INSTITUTIONAL ADVISORY & SOLUTIONS

INSTITUTIONAL GOLD!

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We find that gold has not performed particularly well compared to other assets. However, there is a place for gold-related assets in institutional portfolios separate from commodities and energy equities. The role for gold lies in its diversification and macroeconomic hedging benefits.

We examine the potential role of gold in institutional portfolios, analyzing this question from three perspectives – as a hedge against inflation, a hedge against slow economic growth, and as a portfolio diversifier within a portfolio of financial assets (*e.g.*, stocks and bonds).

Gold's correlation with other financial assets and macroeconomic variables is sensitive to the investor's horizon and time period, which explains why there are often conflicting views on gold in institutional portfolios. We discuss the difficulties of estimating correlations, especially for long horizons. We highlight the importance of measuring estimation uncertainty and how it can be incorporated into the portfolio construction process.

PGIM

Institutional investors already have target allocations to commodities. But, is there a case for a separate allocation to gold? During the last recession (December 2007 – June 2009), a period of high stock market implied volatility (VIX = 35%), equity and commodities returned -35% and -40%, respectively, while gold returned 20%. Fortunately, the damage from such volatility spikes is transitory and most institutional CIOs can safely ride them out.[1] However, an allocation to gold would have softened the short-term drawdown.

We examine the potential role of gold in institutional portfolios, analyzing this question from three perspectives – as a hedge against inflation, a hedge against slow economic growth, and as a portfolio diversifier within a portfolio of financial assets (*e.g.*, stocks and bonds).[2]

There have been many studies – often with contradictory conclusions – regarding the potential institutional role for gold. We show that these inconsistent findings are attributable to differences in the time periods examined and the assumed investment horizon. To assess gold's portfolio role, we use returns from January 1973 to January 2019 and measure correlations using horizons more relevant to CIOs. We find that gold-related assets can play an important role, separate from commodities and energy equities, in institutional portfolios.

We paper is divided into four parts:

- How can institutions invest in gold? The size of the financial gold market excluding official sector holdings is more than $1.9t (Source: World Gold Council). We discuss several institutional vehicles: physical gold, futures, gold miner equity, royalty agreements and streaming agreements. For futures, we illustrate a rolling futures strategy to improve performance for long-term gold exposure. For gold miner equity, we illustrate how an investor might construct an investable earnings quality style portfolio that outperforms the sector.
- What has been the long-term performance of gold and its correlation to other institutional assets and macroeconomic variables? We find that gold has not performed particularly well compared to other assets. The role for gold lies in its diversification and macroeconomic hedging benefits. Gold's correlation with other financial assets and macroeconomic variables is sensitive to the time period and the investor's horizon. For investment horizons relevant to CIOs, we find that gold is somewhat positively correlated with the US CPI (more so than US equity) and negatively correlated with growth (contrary to equity). Gold has exhibited strong negative correlations with equity and Treasuries.
- What is the allocation to gold-related assets in hedging portfolios? We highlight how this allocation depends on both the investor's objective (*e.g.*, protection against inflation or slow growth) and investment horizon. We find that there is a role for gold-related assets separate from commodities and energy equities. While we illustrate gold's role in add-on hedging portfolios, there may be more efficient and better alternatives to achieve a desired portfolio-level macroeconomic sensitivity.
- Finally, we discuss the difficulties of estimating correlations, especially for long horizons. We highlight the importance of measuring estimation uncertainty and show how this uncertainty can be incorporated into the portfolio construction process for long-term portfolio allocations. Although not discussed here, long-horizon investors can alternatively employ dynamic allocation strategies.

1. Institutional Vehicles for Investing in Gold

There are several institutional vehicles to add exposure to gold: physical gold, exchange-listed futures, equity investments in miners, and royalty and streaming agreements.

Physical Gold

Physical gold (bullion, or spot gold) incurs storage and insurance costs (these costs may be offset to some extent by lending fees).[3] Much of the physical gold market is traded over-the-counter *via* London Precious Metal Clearing Ltd. where roughly $27b (approximately 21m ounces) of bullion is cleared daily (as of October 2018).[4]

Investors can select two types of gold accounts with a "bullion bank": an **allocated account** which stores gold in a segregated account, or an **unallocated account** giving the investor a legal right to a certain amount of the bank's total gold holdings. A key difference between the two account types is that an investor with an unallocated account is considered an unsecured bank creditor. However, the bank may lend gold in the unallocated account, thereby providing benefits like reduced storage fees.

1 See A. Xie, "When the Dust Flies… How Volatility Events Affect Asset Class Performance," PGIM IAS, April 2018.
2 H. Parikh and W. Zhang, "Diversity of Real Assets: Portfolio Construction for Institutional Investors," PGIM IAS, June 2019.
3 An indication of bullion storage costs is the COMEX warehousing (allocated storage) cost of $15 per month, per 100oz contract, which translates to about 15bp/y.
4 The Shanghai Gold Exchange is also very active. Average daily trading volume in gold products (as of September 2018) was roughly $12b (approximately 9m ounces).

Alternatively, investors can consider using an exchange-listed gold ETF. For example, one ETF – ticker: GLD – has a relatively large market capitalization of $32b (as of January 2019) with an average daily trading volume of approximately $1b. However, this ETF has a relatively large expense ratio of 40bp/y. GLD shares are backed by physical gold. Large investors, via authorized participants, can create or redeem shares (in baskets of 100,000 shares) for physical gold.

Gold Futures

Instead of physical gold, investors can gain exposure to gold *via* exchange-traded gold futures. This is an actively traded market with daily traded volume on the COMEX (one of several exchanges that lists gold futures) equivalent to about 20m ounces. As of December 2018, open interest was 403,733 contracts (approximately $50b in market value).

Since a position in gold futures requires only a small upfront investment, to compute total returns for gold futures we include a cash return on the futures contract's notional amount. In other words, we treat the futures position as a fully-funded position as would be the case if gold bullion were purchased. The spot vs. futures price differential implicitly reflects storage, financing, lending opportunities and insurance costs. Gross performance of gold futures (with a monthly roll frequency) has been lower than the gross return on spot gold (4.8% vs. 5.2%, since 1978; Figure 2) perhaps reflecting the better liquidity (lower bid-ask spreads and market impact cost) of futures relative to spot markets, the storage cost embedded in gold futures prices but not in the spot price, or that the futures price has often traded above the expected future spot price (contango). Spot and futures returns have had similar annualized volatility (approximately 19.0%) and a monthly total return correlation of 0.98. Given that spot and futures performances are similar an investor may choose a strategy of rolling futures contracts for long-term gold exposure.

Since the futures contract is often in contango a rolling futures strategy has a potential added cost. To manage this cost, we illustrate a strategy that rolls to the cheapest futures delivery month out to six months, based on the implied convenience yield. We first identify the contract month (up to six months expiration) with the highest implied convenience yield ($\boldsymbol{cy_t}$), *i.e.,* the cheapest available futures contract month (Equation 1).[5]

$$cy_t = 1 + rf_t - \sqrt[-(T-t)]{\frac{F_t^{(T)}}{S_t}} \tag{1}$$

where $\boldsymbol{rf_t}$ is risk free rate, $F_t^{(T)}$ is the gold futures price at time t and maturing at time T (maturity ranges from 1m to 6m), and $\boldsymbol{S_t}$ is the spot gold price.

We then hold the futures for one month and roll into the cheapest available futures at the end of the month. However, if the implied convenience yield is negative, we hold bullion instead (using an ETF).[6] For 73% of the months (*i.e.,* 201 months) the strategy holds futures contracts, otherwise ETF shares. While gold and gold futures had 5.5%/y and 5.2%/y returns, respectively, the rules-based gold futures strategy had a 7.1%/y return.[7] The annual volatility of the strategy was 16.2%, matching bullion.

Gold Miner Equity

Investors can also obtain exposure to gold *via* listed equity shares of gold miner companies. The performance of individual equity shares includes exposure to the spot gold price but also exposure to firm-level characteristics like production costs, exploration and development, management expertise, financial leverage and hedging strategy. These characteristics set mining companies apart. Gold miners have the responsibility for decommissioning and covering the reclamation costs of closed mines.[8]

Monthly total returns for gold miner equity sector have a correlation of 0.65 with spot gold returns and a 0.23 correlation with US equity returns. However, the annualized return volatility of gold miner equity (36.4%) has more than twice the return volatility of gold bullion (18.8%) and US equity (14.8%).

5 Convenience yield is the premium for holding a physical asset instead of a contractual exposure – the value of the physical product being immediately available.

6 We use gold bullion performance instead of gold ETF performance due to the limited performance history of gold ETFs. However, gold ETF performance may differ from that of gold bullion by approximately 40bp/y *i.e.,* ETF management expense fees. For the months when the strategy switches to the gold ETF we reduce gold bullion performance by this 40bp/y management fee. We ignore the small (0.01%) bid/ask spread for trading gold ETF shares. The choice of ETF may cause differences in management expense fees and also in transaction costs. See "SPDR Gold Shares ETF Capital Markets Perspective" SSGA and World Gold Council, May 2016.

7 We ignore the COMEX futures transaction costs. Best hourly bid/ask spreads are only $0.1 per a futures contract. See, "Liquidity of COMEX Gold & Silver is Excellent" CME Group, June 2018.

8 C. Louney, "Gold cash costs – An all-in perspective", Barclays Capital, 2014. The modelled average all-in sustaining costs were $941/oz while marginal all-in sustaining costs were $1,285/oz. These costs differ across companies. All-in sustaining costs is an extension of the existing "cash cost" metrics and incorporates costs related to sustaining production.

Since gold miner stocks have historically not performed well, we consider the performance of a subset with higher earnings quality. We illustrate how an investor might construct an earnings quality style portfolio in the global gold miner sector: a market-capitalization weighted portfolio of the top 20% global gold miner companies with the highest earnings quality ranking.[9] Our strategy, which is fully replicable using available contemporaneous information, rebalances this style portfolio monthly. As of November 2018, the stocks in this portfolio had a combined market-cap of $37b. While gold miner equity had a 0.4%/y average annual return, the high-earnings quality gold miner equity portfolio had a 6.4%/y return with a lower volatility (33.8% vs. 37.8%). Due to limited history (only since 1996) we are unable to use the gold miner high-earnings quality style portfolio for our analysis. Instead, we use the gold miner equity sector. We use North American gold miner equity index (Source: Datastream) for gold miner sector returns. Since the correlation of the high-earnings quality style portfolio to the gold miner sector is high ($\rho = 0.92$), inferences in subsequent sections may still be valid for the high earnings-quality portfolio.

Gold Royalty Agreements and Streaming Agreements

Another institutional vehicle for investing in gold are **royalty agreements**.[10] In return for an upfront payment to a miner, a royalty agreement gives the investor an interest in the mine's future production. There are a variety of royalty agreements but they typically share the following characteristics:

- Are not subject to cash calls to fund exploration, development, capital, environmental or closure costs;
- Provide exposure to the upside of commodity price, mineral reserves and production increases;
- May provide participation interest in new discoveries; and
- Do not involve operational or development management.

There are two types of royalty agreements. *Revenue-based royalties* are based on the net smelter return (NSR) which is the miner's net revenue from the mine's output after transportation and refining costs. The agreement specifies the fraction of the NSR that the mine operator must pay to the agreement holder. An NSR royalty may be structured so that it terminates after a certain amount of gold production or may last for the life of the mine.

Profit-based interest royalties are based on the operating profit as defined in the royalty contract. Typically, royalty payments begin only after the miner has recovered capital costs. A net profit interest (NPI) royalty is the most common of this type of royalty agreement.

Figure 1: Gold Royalty Companies
(as of November 2018)

Ticker	Exchange	Name	Listing Date
FNV	Toronto/NYSE	FRANCO-NEVADA	12/3/2007
OR	Toronto/NYSE	OSISKO GOLD ROYALTIES	6/2/2014
WPM	Toronto/NYSE	WHEATON PRECIOUS METALS	3/8/1995
SSL/SAND	Toronto/NYSE	SANDSTORM GOLD	8/13/2007
RGL/RGLD	Toronto/NASDAQ	ROYAL GOLD	6/9/1981

Note: Wheaton Precious Metals is a precious metal streaming company. Individual security names are provided for illustration purposes only and should not be construed as investment advice regarding particular securities or an offer or solicitation to buy or sell particular securities.

Source: Datastream.

9 The earnings quality factor (as defined by S&P Capital IQ) is an equally-weighted average of five quality factors – cash conversion cycle, net profit margin, working capital accruals, accrual ratio, and net income stability. Cash conversion cycle is the sum of average receivable collection period and average inventory processing period, minus payables payment period. Net profit margin is the ratio of trailing four quarter income before extraordinary items to trailing four quarter sales. Working capital accruals is the change from four quarters ago in non-cash assets, minus the change in current liabilities (excluding short term debt and taxes payable) and minus depreciation, divided by average total assets over the past year. The cash flow-based accrual ratio uses the trailing twelve-month difference between net income and cash flows (operation and investment) as the accrual measure. Net income stability is the ratio of 5-year average of the one-year percentage change in net income over the mean absolute deviation in the one-year percentage change in net income going back five years.

10 N. Mordant, and E. Rocha. "Private equity, pension funds eye more metal streaming deals" Reuters, 2015. The authors discuss that in 2015 there was more than $4.5b worth of streaming deals. D. Giemajner, 2017. Royalty and streaming agreement definitions source: Franco-Nevada. "Glencore and Ontario Teachers' Pension Plan announce the creation of a new partnership focused on base metals streams and royalties," *Ontario Teachers' Pension Plan, 2017.*

Figure 2: Gold & Other Asset Class Performance

(USD Annualized Total Returns; January 1973 – January 2019)

	Gold (bullion)	Gold Futures	Gold Miner Equity	Gold Futures Strategy	Gold Miner Equity (High Quality)	US Equity	US Energy Equity	GSCI	Treasury	TIPS	Cash	US Dollar
Since 1973	—	—	—	—	—	—	—	—	—	—	—	—
Return	6.8%	—	6.4%	—	—	10.4%	10.4%	5.6%	7.0%	7.0%	4.8%	–0.3%
Volatility	20.8%	—	35.2%	—	—	15.2%	19.3%	20.3%	5.1%	6.1%	1.1%	8.8%
Sharpe Ratio	0.09	—	0.04	—	—	0.36	0.29	0.04	0.42	0.36	—	–0.58
Correlation	—	—	0.57	—	—	–0.01	0.10	0.26	0.05	0.16	0.03	–0.35
Since 1978	—	—	—	—	—	—	—	—	—	—	—	—
Return	5.2%	4.8%	6.5%	—	—	11.9%	10.9%	4.5%	7.0%	6.5%	4.6%	0.0%
Volatility	18.8%	19.0%	36.4%	—	—	14.8%	19.4%	19.3%	5.3%	6.2%	1.1%	8.9%
Sharpe Ratio	0.03	0.01	0.05	—	—	0.49	0.32	–0.01	0.45	0.30	—	–0.52
Correlation	—	0.98	0.65	—	—	0.00	0.15	0.25	0.05	0.19	0.01	–0.37
Since 1996	—	—	—	—	—	—	—	—	—	—	—	—
Return	5.5%	5.2%	0.4%	7.0%	6.4%	8.9%	8.5%	–0.4%	4.6%	5.0%	2.3%	0.5%
Volatility	16.1%	16.4%	37.8%	16.2%	33.8%	15.2%	19.7%	22.0%	4.3%	5.4%	0.6%	7.9%
Sharpe Ratio	0.20	0.18	–0.05	0.29	0.12	0.44	0.32	–0.12	0.54	0.51	—	–0.22
Correlation	—	0.99	0.76	0.99	0.74	0.01	0.15	0.25	0.21	0.38	–0.04	–0.40

Note: Correlations are to bullion. Gold futures (S&P GSCI Gold Index) data are available from January 1978. See Appendix for asset class definitions and mappings.

Source: PGIM IAS, St. Louis Federal Reserve (FRED) and Datastream. For illustrative purposes only. Past performance is not a guarantee or reliable indicator of future results.

Institutional investors can also enter into **streaming agreements** which are metal purchase agreements that provide, in exchange for an upfront payment, the right to purchase at a preset price all, or a portion, of a mine's annual production. Streaming agreements are not royalty agreements because there are ongoing cash payments to purchase the mine's output. They can have a term equal to the mine's lifetime or a fixed term (say, 25y).

Streaming agreements can be highly customized. For example, a mine owner may agree, in exchange for an upfront payment, to sell to the agreement holder the equivalent of 7.5% of a mine owner's interest in the gold produced until 990,000 ounces of gold have been delivered, and 3.75% thereafter. The agreement holder's ongoing cash payment to the mine owner is tied to the prevailing spot price: say 30% of the prevailing spot price for the first 550,000 ounces of gold, 60% thereafter.[11]

Given the customized and bilateral nature of these agreements, price and trading data are limited. To proxy the performance of this gold asset type we initially considered using the returns on listed royalty companies which are businesses which serve as conduits for investors to gain exposure to royalty and streaming agreements. Such companies have relatively steady cashflows, few employees and low operating costs. Some royalty companies can also have a direct ownership interest in mining operations.[12]

However, there are few royalty companies within the gold miner industry equity universe, and even fewer with a long time series of returns (Figure 1). Royal Gold, domiciled in the US, has the longest history in the publicly-listed market (since 1981) and currently has the largest market-capitalization of all royalty companies ($6.2b, May 2019).[13] We considered using Royal Gold as a long-term proxy for gold royalty and streaming agreements, but a single stock would include a significant component of idiosyncratic risk and would not be representative of a large institutional investor's exposure to royalty agreements. Since royalty companies have good earnings quality, an investor could consider using the high-earnings quality miner portfolio as a royalty proxy.

11 https://www.barrick.com/news/news-details/2015/Barrick-Announces-Streaming-Agreement-With-Royal-Gold/default.aspx

12 For example, Osisko Gold Royalties Ltd. assets include 5% NSR on the Canadian Malartic as well as 15.5% ownership interest in Osisko Mining Inc., a 33.4% interest in Barkerville Gold Mines Ltd. and a 12.7% interest in Falco Resources Ltd.

13 Individual stock names are provided for illustration purposes only and should not be construed as investment advice regarding securities or an offer or solicitation to buy or sell.

2. Long-term Gold Performance

Jensen *et al*. [2018] report that bullion funds had better performance than gold miner equity funds, but this was over a relatively short period (January 2007 – December 2016).[14] Over a longer period, January 1973 to January 2019, gold (6.8%/y) and gold miner equity (6.4%/y) outperformed the US Dollar index (DXY) (-0.3%/y), commodity (GSCI) (5.6%/y) and cash (4.8%/y). Nevertheless, gold trailed other important institutional assets (*e.g.,* US equity (10.4%/y) and Treasury (7.0%/y)); with significantly higher volatility.

Overall, gold has not demonstrated strong long-term performance compared to other institutional asset classes. The portfolio argument for gold lies in its diversification and hedging potential.[15]

Estimating Gold's Sensitivities to Macroeconomic and Market Variables

We estimate the Spearman correlations of gold to US financial assets (US equity and Treasury bonds) and to macroeconomic variables (US CPI and the level of economic activity (Chicago Fed National Activity Index, CFNAI)).[16] We calculate correlations using 6m up to 5y horizon returns. Our data sample is monthly from January 1973 – January 2019. As discussed in Section 4, constructing long horizon returns reduces the number of observations, reducing the precision of the correlation estimate.[17]

To minimize the potential that the starting month will affect the estimated correlation, we use *overlapping* returns. Since there is only one data sample, we know that correlation estimates have sampling error. In other words, if we had another historical data sample we would likely generate a different correlation estimate. As discussed in Section 4, we can estimate this sampling error using bootstrap methods by randomly sampling (with replacement) from the given data sample to generate another sample and another correlation estimate. We repeat this sampling 10,000 times, producing 10,000 correlation estimates for a given return horizon. Figure 3 reports the average of these 10,000 correlation estimates. The advantage of this bootstrap method is that we can measure the range of correlation estimates (Figures 4 and 5).

For correlations to the CPI we see wide variation across both assets and horizons. Gold's average correlation with the CPI was positive, but not particularly large. Importantly, however, the average correlation increased with the length of the return horizon (6m $\rho = 0.10$; 5y $\rho = 0.21$). The GSCI, TIPS, gold miner equity and US energy equity had larger correlations to the CPI which also increased with the return horizon. For example, the average 6m correlation of gold miner equity with CPI was 0.11 but the 5y correlation was 0.48. Notably, cash and Treasury also had high 5y correlations to the CPI. This is because at longer horizons actual inflation is similar to expected inflation which is embedded in the yield of fixed income assets. US equity tended to have negative to low positive average correlation to the CPI (6m $\rho = -0.16$; 5y $\rho = 0.08$) which may be surprising to investors who believe that equities have reliable inflation hedging properties.

Our findings differ from Jensen *et al*. [2018] who found that bullion funds were insignificantly correlated ($\rho = 0.04$) to expected inflation. The difference with our findings could be due to Jensen's shorter data period, the much shorter return horizon (weekly), and/or differences between their expected inflation proxy and actual inflation. CIOs might be more interested in asset class correlations with actual rather than expected inflation. For example, for public plan sponsors their liabilities are linked to actual inflation.

For average correlations to CFNAI, we observe that gold, gold miner equity, TIPS and Treasury had negative or low positive correlations. While the correlations generally increased with the horizon, gold's correlation to CFNAI became more negative. The 6m correlation of gold to CFNAI was -0.09, but the 5y correlation was -0.21, suggesting that gold might serve as a long-term hedge against economic slowdowns. As expected, US equity and energy equity had the highest (positive) average correlations to CFNAI (5y, $\rho = 0.45$ and 0.47, respectively).

On average, gold, gold miner equity, GSCI and TIPS had very low (negative) 5y horizon correlations to US equity. Therefore, these assets can be considered as diversifying to US equity. In fact, gold is the only asset to have negative correlation to equity, at the 90th percentile, for 5y horizon returns.

Of all the assets, gold was most diversifying to Treasuries due to the negative correlation (6m, $\rho = -0.03$) which became more negative as the horizon increased (5y $\rho = -0.28$).

Figures 4 and 5 (using the bootstrapped data underlying Figure 3) summarize the differences in the average correlation and its variability across the bootstrapped samples, depending on the length of the return horizon (6m and 5y). For example, at the longer horizon gold is more positive correlated to the CPI and more negatively correlated to CFNAI, equities and Treasuries. Importantly, however, at the longer horizon the interval bands (10th and 90th percentiles) for the correlation estimate are wider. For example, across the 10,000 bootstrapped samples gold's estimated 5y correlation to the CPI ranges from -0.26 (10th percentile) to +0.65 (90th percentile). This reveals the difficulty facing the CIO – what is the relevant correlation value to assume for portfolio construction?

14 See G. R. Jensen, R. R. Johnson, and K. M. Washer, "All That's Gold Does Not Glitter," *Financial Analysts Journal,* Vol. 74(1): 59-76, 2018.
15 See P. Samuelson, "The Long-Term Case for Equities – And How It Can Be Oversold," *Journal of Portfolio Management,* Vol. 21(1), 1994.
16 https://www.chicagofed.org/research/data/cfnai/current-data. See Section 4 for a discussion of the Spearman correlation.
17 Section 4 describes our empirical methodology in detail.

Figure 3: Correlation of Gold to Various Assets and Macroeconomic Variables, by Length of Horizon Returns
(1m to 5y Horizons, Spearman Correlations; USD Total Returns; January 1973 – January 2019)

Horizon	Gold	Gold Miner Equity	US Equity	US Energy Equity	GSCI	Treasury	TIPS	Cash
to CPI								
1m	0.13	0.03	–0.12	0.01	0.13	–0.04	0.08	0.44
6m	0.10	0.11	–0.16	0.16	0.42	0.01	0.21	0.63
1y	0.13	0.15	–0.14	0.22	0.48	0.13	0.34	0.70
2y	0.10	0.19	–0.07	0.31	0.50	0.29	0.44	0.77
3y	0.12	0.31	0.03	0.40	0.55	0.39	0.52	0.80
5y	0.21	0.48	0.08	0.41	0.61	0.51	0.61	0.80
to CFNAI								
1m	–0.03	–0.03	–0.04	0.00	0.03	–0.13	–0.14	0.05
6m	–0.09	–0.08	0.06	0.12	0.12	–0.23	–0.32	0.07
1y	–0.07	–0.08	0.09	0.20	0.15	–0.23	–0.37	0.07
2y	–0.07	–0.02	0.27	0.40	0.18	–0.13	–0.34	0.08
3y	–0.13	–0.01	0.41	0.49	0.21	–0.04	–0.25	0.17
5y	–0.21	0.01	0.45	0.47	0.23	0.14	–0.07	0.33
to US Equity								
1m	–0.07	0.20	–	0.59	0.10	0.09	0.10	0.04
6m	–0.11	0.21	–	0.56	0.00	0.00	0.00	0.05
1y	–0.20	0.17	–	0.55	0.04	0.08	0.03	0.10
2y	–0.34	–0.05	–	0.50	0.06	0.10	–0.04	0.15
3y	–0.48	–0.16	–	0.45	0.03	0.20	–0.08	0.23
5y	–0.52	–0.11	–	0.37	–0.01	0.35	–0.08	0.31
to Treasury								
1m	0.04	0.06	0.09	–0.04	–0.10	–	0.81	0.23
6m	–0.03	0.04	0.00	–0.15	–0.18	–	0.74	0.40
1y	–0.15	0.03	0.08	–0.08	–0.12	–	0.71	0.47
2y	–0.26	0.03	0.10	–0.08	0.00	–	0.70	0.59
3y	–0.30	0.06	0.20	–0.03	0.13	–	0.68	0.66
5y	–0.28	0.23	0.35	0.13	0.33	–	0.64	0.72

Note: Spearman correlations are average values across 10,000 samples. 1m Spearman correlations are reported for reference using all available data. For 5y, CFNAI is a 5y moving average of monthly CFNAI. 5y |ΔVIX| measures 5y moving average of absolute monthly change in the VIX level. See Appendix for asset class definitions.
Source: PGIM IAS. For illustrative purposes only.

Figures 4 and 5 offer useful information for CIOs. Judging by the range of correlation estimates, GSCI, gold miner equity and cash might be more reliably correlated with the CPI compared to gold and equities. For long-term positive correlation to growth, US equity and cash (and to a lesser extent GSCI) seem to be more reliable than gold. In contrast, gold is the most reliable long-term hedge for both equities and Treasuries while the GSCI is an ambivalent long-term hedge for equities and is positively correlated with Treasuries.

Gold is often regarded as a "fear hedge." However, we know that VIX shocks (our fear event proxy) are transitory, as are the associated equity market drawdowns.[18] For longer horizons does gold have a role to play as a fear hedge? We calculate correlations to changes in VIX (not reported). We use the average of the absolute monthly changes in VIX (|ΔVIX|) over the return horizon as the fear variable.

18 See A. Xie, PGIM, 2018.

Figure 4A: Correlations to CPI
(USD Total Returns; January 1973 – January 2019)

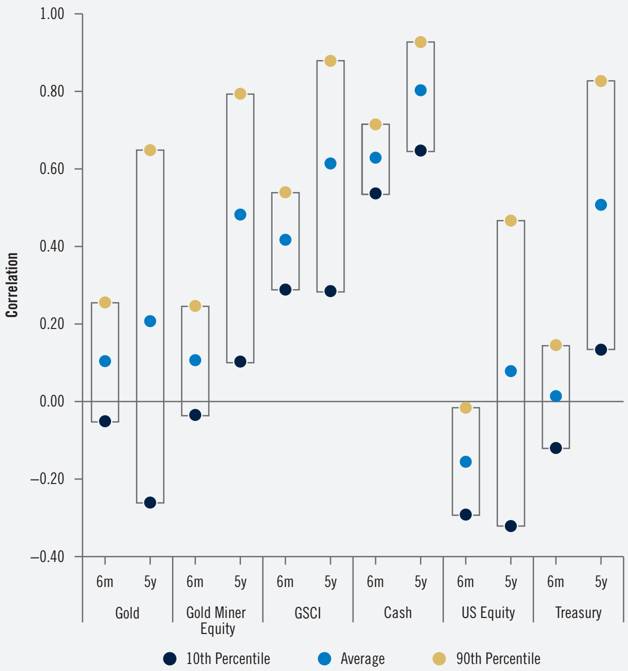
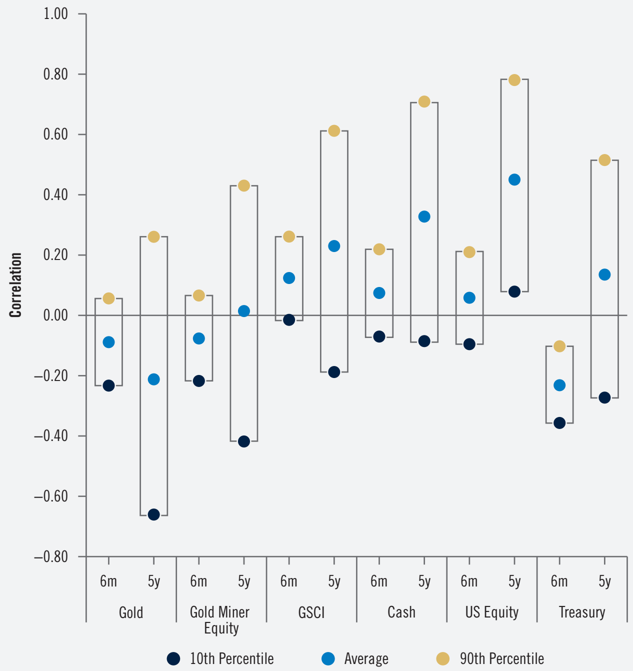


Figure 4B: Correlations to CFNAI
(USD Total Returns; January 1973 – January 2019)



Source: PGIM IAS. For illustrative purposes only.

Figure 5A: Correlations to US Equity
(USD Total Returns; January 1973 – January 2019)

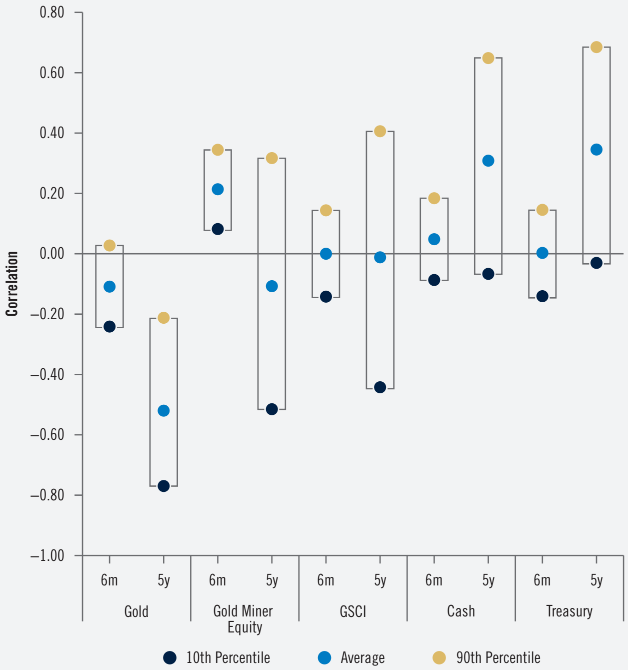
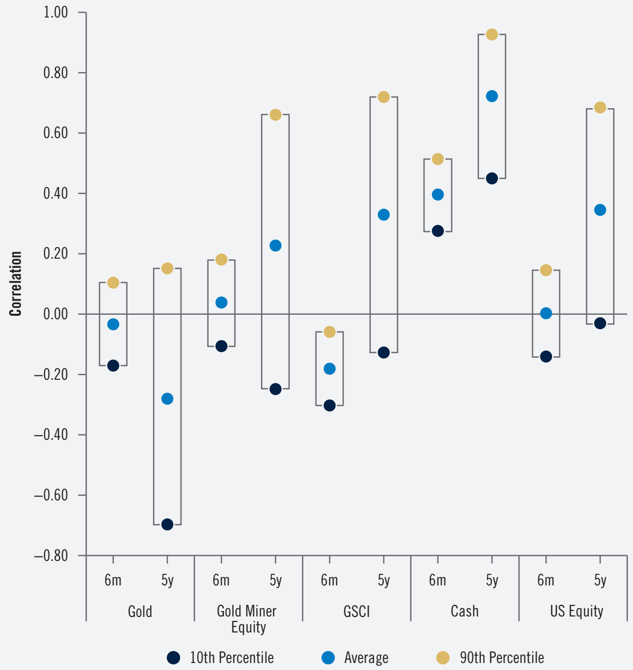


Figure 5B: Correlations to Treasury
(USD Total Returns; January 1973 – January 2019)



Source: PGIM IAS. For illustrative purposes only.

Figure 6: Asset Class Sensitivities to Macroeconomic and Market Variables

(Multivariate Regressions, Average Betas and R^2; USD Total Returns; January 1973 – January 2019)

		Average Betas and R^2, by Return Horizon				
		6m	1y	2y	3y	5y
	c	**–0.07**	**–0.14**	–0.09	–0.05	–0.02
	DXY	**–0.93**	**–1.06**	**–1.25**	**–1.25**	**–1.22**
	$\lvert\Delta VIX\rvert$	**2.13**	4.09	3.42	2.06	0.84
Gold	Δeff_FF	0.06	**0.43**	**0.68**	**0.75**	**0.79**
	CPI	**3.08**	**3.23**	**1.88**	**1.56**	**1.40**
	CFNAI	–0.01	–0.06	**–0.09**	**–0.13**	**–0.19**
	R^2	0.30	0.43	0.56	0.60	0.66
	c	0.01	0.00	0.00	0.03	0.05
	DXY	**–0.91**	**–1.06**	**–1.52**	**–1.83**	**–2.20**
	$\lvert\Delta VIX\rvert$	0.32	1.06	–0.07	–2.10	–4.48
Gold Miner Equity	Δeff_FF	–0.04	–0.07	0.18	0.09	–0.30
	CPI	1.72	2.29	1.92	**2.36**	**2.93**
	CFNAI	0.00	–0.01	–0.06	–0.09	–0.10
	R^2	0.09	0.11	0.27	0.32	0.40
	c	0.00	–0.01	–0.01	–0.04	–0.04
	DXY	**–0.65**	**–0.69**	**–0.88**	**–1.06**	**–1.40**
	$\lvert\Delta VIX\rvert$	–1.13	–0.97	–1.03	0.32	–0.05
GSCI	Δeff_FF	0.03	0.07	0.09	0.01	–0.49
	CPI	**3.94**	**3.27**	**2.75**	**2.60**	**2.77**
	CFNAI	0.02	0.05	0.02	0.04	0.09
	R^2	0.30	0.28	0.29	0.33	0.42
	c	**0.02**	**0.03**	0.03	0.02	0.01
	DXY	**–0.15**	–0.11	**–0.17**	**–0.19**	**–0.22**
	$\lvert\Delta VIX\rvert$	0.09	0.09	0.21	0.61	0.77
TIPS	Δeff_FF	**–0.09**	**–0.21**	**–0.29**	**–0.30**	**–0.38**
	CPI	**0.89**	**0.91**	**0.98**	**1.01**	**0.95**
	CFNAI	0.00	0.00	0.01	0.01	0.02
	R^2	0.41	0.42	0.61	0.66	0.74
	c	**0.14**	**0.27**	**0.23**	**0.20**	**0.17**
	DXY	0.00	0.01	0.11	0.43	**0.59**
	$\lvert\Delta VIX\rvert$	**–2.70**	**–5.26**	**–4.31**	–2.95	–1.80
US Equity	Δeff_FF	–0.10	–0.23	–0.19	–0.10	0.00
	CPI	–0.56	–0.43	–0.21	–0.31	–0.13
	CFNAI	0.01	0.04	**0.08**	**0.11**	**0.15**
	R^2	0.21	0.23	0.26	0.29	0.28
	c	**0.02**	**0.05**	**0.04**	0.03	**0.02**
	DXY	–0.02	0.02	–0.02	–0.04	–0.03
	$\lvert\Delta VIX\rvert$	0.33	0.20	0.25	0.46	0.58
Treasury	Δeff_FF	**–0.08**	**–0.25**	**–0.36**	**–0.39**	**–0.47**
	CPI	0.12	0.37	**0.57**	**0.66**	**0.87**
	CFNAI	0.00	0.02	**0.04**	**0.05**	**0.05**
	R^2	0.26	0.32	0.51	0.58	0.63

Note: Coefficients are average values across 10,000 samples. Coefficients are in bold if statistically significant at a 90% confidence level.
Source: PGIM IAS. For illustrative purposes only.

Figure 7: Summary – Gold and Gold Miner Correlations with CPI and CFNAI; Short vs. Long Return Horizons

	Short Horizon	Long Horizon
Gold vs. CPI	• Low, but positive correlation • Higher correlation than US equity and Treasury	• Correlations increase with increases in the return horizon • Higher correlation than US equity, but lower than Treasury
Gold vs. CFNAI	• Low and negative correlation • Lower correlation than US equity, but higher than Treasury	• Gold is the only asset with negative correlation
Gold Miner Equity vs. CPI	• Low, but positive correlation • Higher correlation than US equity and similar to gold	• Higher correlation to CPI • Higher correlation than gold and US equity and similar to Treasury
Gold Miner Equity vs. CFNAI	• Low and negative correlation • Lower correlation than US equity, but higher than Treasury	• No correlation to CFNAI • Lower correlation than US equity and Treasury

Source: PGIM IAS. For illustrative purposes only.

This variable is akin to VIX volatility and measures directional uncertainty in equity markets. This way sharp VIX movements can be distinguished from small up and down changes. Jensen *et al.* [2018] did not find gold funds to be a hedge against market uncertainty. In contrast, our correlation analysis suggests that for horizons up to 5y gold returns had positive correlation to $|\Delta VIX|$ (5y $\rho = 0.29$), suggesting some hedge against market uncertainty. In contrast, US equity correlation to $|\Delta VIX|$ was negative across all investment horizons (5y, $\rho = -0.34$).

We next use *multivariate* regression to estimate the relationship of various asset class returns to macroeconomic and financial market variables (Figure 6). Like our correlation estimation methodology, we use the bootstrap to conduct the regression analysis using simultaneously sampled asset returns, macroeconomic and market variables. We repeat the analysis 10,000 times, each time with a different sample, for each set of horizon returns from 6m to 5y. With the large number of samples, we can estimate a sample standard error for the estimated betas which we use when constructing hedging portfolios. We also report average adjusted-R^2s and t-statistics.[19]

Regardless of the return horizon, gold persistently had a negative and statistically significant exposure to US Dollar returns DXY (*i.e.,* gold falls when the dollar strengthens). While positive and statistically significant, gold betas to CPI tended to decrease, and move closer to 1, as the return horizon increases (6m, 3.1; 5y, 1.4). Gold exposures to CFNAI are negative for all horizons, but only become statistically significant for return horizons of 1y or longer.

The $|\Delta VIX|$ betas to gold are positive at all horizons, but only statistically significant at 6m. This suggests gold mostly as a short-term fear hedge. For longer horizons (2y or longer) gold also has statistically significant and positive betas to the effective Fed Funds rate (Δeff_FF). These relationships can be time varying (see Appendix for how the estimated betas varied in the sub-sample).

For gold miner equity, the CPI betas are positive across all horizons (statistically significant at 3y and 5y horizons) and increase with the horizon. The 5y CPI beta is the highest relative to other assets, including energy equity (not shown). This suggests that gold miner equity has been a good long-term inflation hedge. Only gold and gold miner equity had negative exposures to CFNAI, suggesting these assets did better in low growth environments.

19 We use Newey-West standard errors to adjust for both autocorrelation and heteroskedasticity.

US equity, across all return horizons, had positive exposures to economic activity. The exposures were statistically significant from 2y to 5y. US Equity has negative exposures to $|\Delta VIX|$, but only statistically significant up to 2y.[20]

Across all return horizons, GSCI had statistically significant, positive betas to CPI, and negative betas to DXY. However, like gold, its betas to CPI decreased as the return horizon increased (6m, 3.94; 5y, 2.77).

Figure 7 summarizes the gold and gold miner equity relationships with CPI and CFNAI, and how these relationships differ with the length of the investment horizon.

3. Gold-related Assets in Hedging Portfolios

Investors may want to add a hedging portfolio that targets a high inflation beta (for inflation protection) or a low growth beta (for low-growth protection). There may be more efficient and better alternatives to achieve a desired overall portfolio-level macroeconomic sensitivity, but some investors may prefer to allocate to an add-on hedging portfolio to achieve an overall portfolio sensitivity without disturbing the main portfolio.

Investors can construct hedging portfolios with an overall target beta to a specific macroeconomic variable, but penalize the allocation weighting to assets whose beta estimates are less reliable. Using bootstrap methods, we estimate in Section 2 the distribution of betas (means and standard errors) to macroeconomic and market variables for different horizons.[21] We use these beta estimates to construct hedging portfolios.

Suppose an investor wants a hedging portfolio that provides inflation protection at a 5y horizon such that the target inflation beta is 2.0. The investor can perform the following optimization:

$$\begin{aligned} &\min \frac{1}{2} w' V w \text{ where } V = \text{Diag}\left(\sigma_{\beta_{\text{CPI\_5y}}}{}^2\right) \\ &\text{s.t. } w'\bar{\beta}_{\text{CPI\_5y}} = 2.0\,, w'i = 1 \text{ and } w \geq 0 \end{aligned} \tag{2}$$

where $\boldsymbol{w}$ is the vector of asset weights in the hedging portfolio, $\boldsymbol{V}$ is the covariance matrix with each diagonal element equal to the variance of the $\beta_{\text{CPI\_5y}}$ for each asset and $\beta_{\text{CPI\_5y}}$ is the vector of average $\beta_{\text{CPI\_5y}}$ for all assets. Investors may have other objectives than minimizing the variance of the beta estimate for the hedging portfolio but here we show the impact of beta uncertainty on portfolio weights.

Figure 8 shows optimal portfolio weights for different target CPI betas, at both a 5y and 6m horizon. For example, the portfolio with a target 5y CPI beta of 2.0, has 11% in gold, 15% in gold miner equity, 32% in GSCI and 42% in cash. Generally, for a higher target portfolio CPI beta the allocations to cash and TIPS decrease, and allocations to gold, gold miner equity and GSCI increase. At a 5y horizon, the portfolio has an allocation to Treasury but only at lower target CPI betas (1.0 and 1.5) because the average 5y CPI beta for Treasury was 0.87 which is lower than the 5y CPI beta for TIPS (0.95).

In contrast the optimal portfolio with a target 6m CPI beta of 2.0, has 10% in gold, 3% in gold miner equity, 9% in energy equity, 28% in GSCI, 19% in TIPS, and 31% in cash.

An investor wishing to construct a portfolio that provides protection to low growth at a 5y horizon, such that the target CFNAI beta is 0.0, can conduct a similar optimization as Equation 2 but substitute CFNAI_5y for CPI_5y betas and volatilities. Figure 9 compares optimal portfolios weights for various target 5y CFNAI betas with those for various target 6m betas. For example, the portfolio with a target 5y CFNAI beta of 0.0, has 12% in gold, 4% in gold miner equity, 10% in Treasury, 50% in TIPS and 24% in cash. At increasingly negative target levels of CFNAI beta (*i.e.,* provide more slow growth protection), allocations to cash and TIPS decrease while allocations to gold and gold miner equity increase.

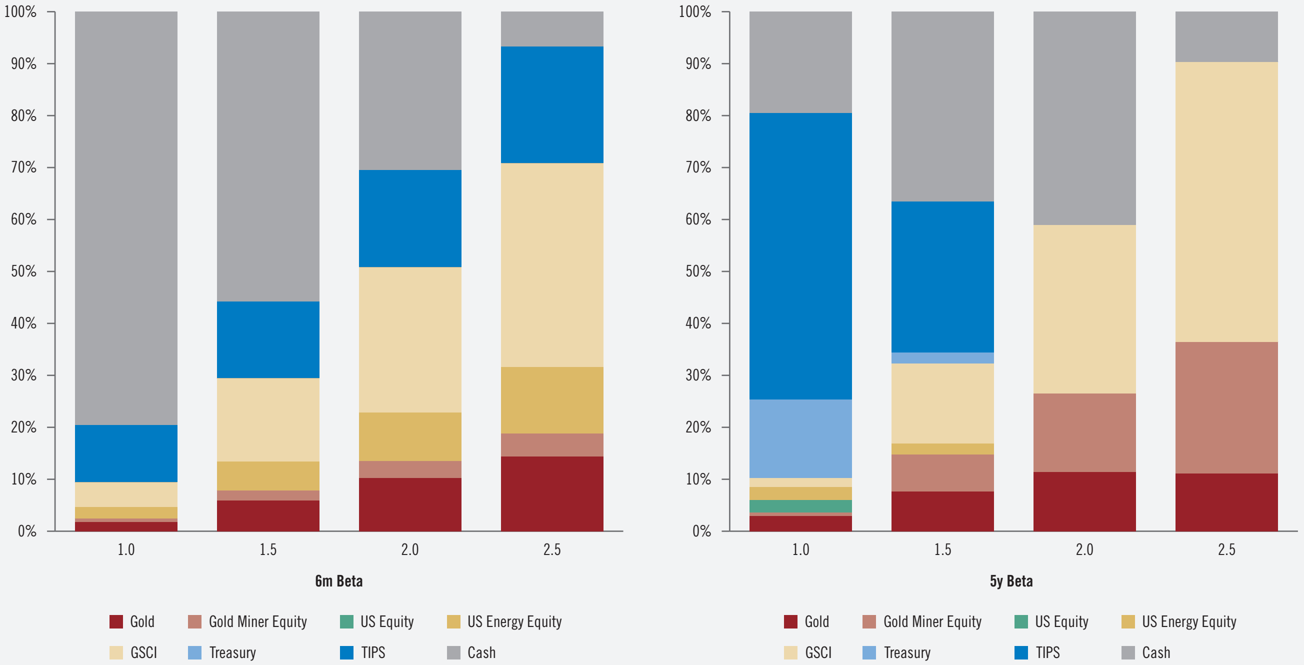
For investors with a short horizon (say 6m) and a target CFNAI exposure of 0.0 the portfolio has much higher allocation to cash (47%) and lower allocation to TIPS (42%), Treasury (5%), gold (5%) and gold miner equity (1%) compared to a portfolio for an investor with a long horizon (5y).

Depending on the investment horizon the optimal weights to asset classes vary for a given target level of inflation or growth exposure. Notably, there is a portfolio role for gold-related assets separate from commodities and energy equities. For our analysis we include eight different asset classes to construct hedging portfolios. Investors can consider including other assets like hedge fund strategies.

20 This finding augments our earlier work (Xie (2018)) demonstrating that while equity sells off in the first few months of a VIX spike, it tends to recover to pre-spike levels in about nine months after the spike.

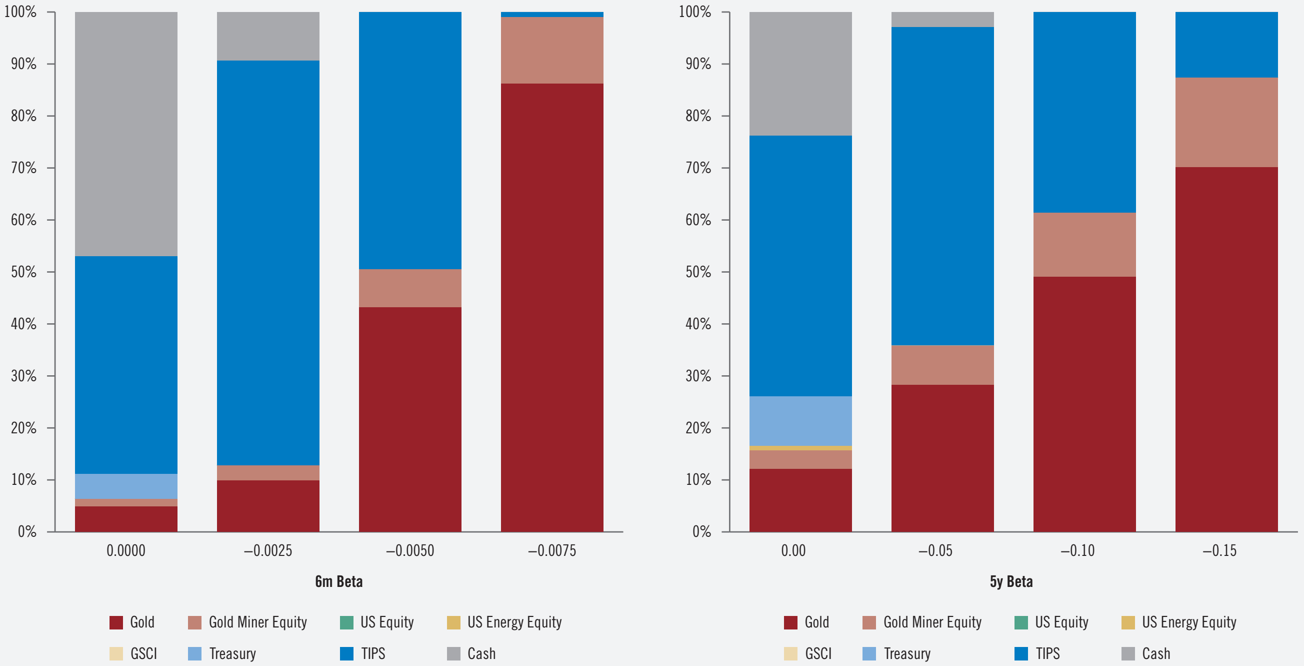
21 See Appendix for standard error estimates.

Figure 8: Inflation Hedging Portfolio Weights – Target 6m vs. 5y CPI Exposures
(USD Total Returns; January 1973 – January 2019)



Source: PGIM IAS. For illustrative purposes only.

Figure 9: Low-growth Hedging Portfolio Weights – Target 6m vs. 5y CFNAI Exposures
(USD Total Returns; January 1973 – January 2019)



Source: PGIM IAS. For illustrative purposes only.

4. Asset Correlations & Investment Horizon

Gold's portfolio role typically hinges on the correlation of its returns with those of other asset returns or macroeconomic variables. However, the literature on the portfolio role of gold is mixed.[22] For example, Jensen *et al*. [2018] report that gold had insignificant correlation to the expected inflation rate. However, their evaluation was over a relatively short time window (January 2007 – December 2016) and they used short horizon (weekly) returns. In contrast, Shapiro and Thomas [2011] show that gold has successfully hedged inflation when viewed over long horizons.[23]

CIOs may be justifiably confused by these conflicting conclusions. In fact, researchers can report widely different correlation estimates depending on the return horizon *even when using the same data sample*. Contributing to the confusion is that researchers sometimes neglect to highlight the uncertainty surrounding their correlation estimates. We discuss these estimation issues to help CIOs better evaluate the portfolio role for gold.

Impact of Sample Size on Correlation Estimation

Estimating the true correlation value using only a sample of data involves sampling error as the estimated value derived from the sample will likely differ from the true (population) value. The magnitude of this error depends in large part on the sample size. To illustrate, suppose we have two time series variables, both Normally distributed ($\mu = 0$ and $\sigma = 0.1$) with no serial correlation.[24] Furthermore, the two variables have a joint Normal distribution with a correlation of -0.1 which we assume we do not know and are trying to estimate.

Suppose we randomly draw 600 paired-observations (which can be interpreted as 50y of monthly returns) from this joint distribution and calculate the sample correlation. This correlation value is unlikely to equal -0.1 due to sampling error. To appreciate the magnitude of this error, we repeatedly draw 600 observation sets from the joint distribution. Repeating 10,000 times we obtain a 99% confidence interval for the sample correlation of [-0.20, 0.00] suggesting that the true correlation is negative (Figure 10).

A smaller sample size, say 120 observations (10y of monthly returns) rather than 600, produces a wider 99% confidence interval [-0.33, 0.14] across 10,000 samples. Even though the underlying true correlation is -0.1, depending on which single estimate is produced the investor could reach a different conclusion regarding the sign of the correlation coefficient – with implications for asset allocation.

We could have used the actual parameters of the known joint distribution (rather than simulation) to mathematically calculate a 99% confidence interval [-0.2, 0.0] for the true correlation value (assuming 600 observations). The benefit of a simulation approach is that it shows possible extreme values. An investor should consider the range of possible values, not just the single estimated value from a single sample. Is the correlation reliably negative or positive? Unfortunately, for financial data we only have one sample of returns *and* we do not know the true underlying statistical process. However, we can still construct confidence intervals. Before doing so, we discuss how, given limited time series data, the investor's return horizon can affect the estimation of the correlation value.

Impact of Return Horizon on Correlation Estimation

Investors may have different investment horizons. A desk trader may have a horizon of a few hours or days. A short-term trend follower may have a horizon of a few weeks. In contrast, an institutional investor like a plan sponsor with long-dated liabilities or a sovereign wealth fund with a goal of intergenerational wealth transfer may have an investment horizon measured in years.

Given a fixed set of time series data, the length of the return horizon affects the precision of the correlation estimate.[25] To illustrate, suppose the investor has 100 observations of 6m returns for two assets spanning 50y. However, an investor may believe that 3y correlations are more relevant given their return horizon. The investor has several choices for estimating the 3y correlation: 1) Use the 100 6m returns, calculate a correlation, and use this value as the estimate for the 3y correlation; 2) Start at the beginning of the series and construct 16 non-overlapping 3y returns and calculate a 3y correlation. The large drop in sample size suggests higher sampling error. Also, the result may be sensitive to the starting period; 3) Generate overlapping 3y returns, essentially starting at each of the possible first six 6m starting periods (producing a sample size of 95, comparable to using 6m returns) and calculate a correlation using these 95 pairs of observations.

We show how the precision of the 3y correlation estimate depends on the investor's choice. As before, we assume the true 6m (or, 3y) correlation between the two time-series ($\mu = 0$ and $\sigma = 0.1$) is -0.1. We randomly sample 100 6m returns from the joint distribution (labelling this sample "s_1") and estimate a correlation value of -0.08. Suppose we use the same sample s_1 and link six consecutive 6m returns to form a 3y return starting at the first 6m period. We then move to the second 6m period and link the next six consecutive 6m returns to form another 3y return, etc. We now have a time series of 95 overlapping 3y returns. It is rather surprising that the estimated 3y correlation from the *same sample* (s_1) is 0.12.[26]

22 A. Ang, *Asset Management: A Systematic Approach to Factor Investing,* Oxford University Press, 2014.

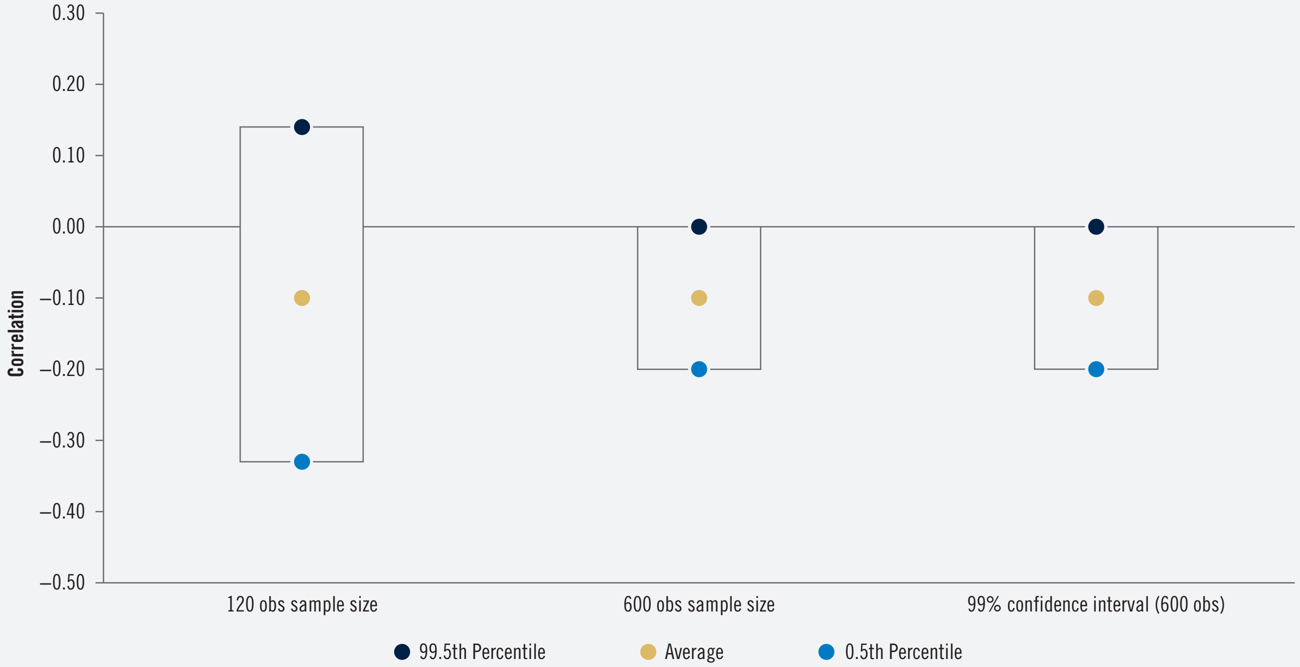
23 See R. Shapiro and R. Thomas, "Should Institutional Investors Own Gold?" *Journal of Index Investing,* Vol 2(3): 19-27, 2011. Authors used non-overlapping, quarterly, 1y and 3y horizon returns and measure correlation of gold and other assets with the CPI for the period from 1978 – 2010.

24 No serial correlation means that this period's return is uncorrelated with any prior return.

25 See R. Goldsticker. "Visualizing the Time Series Behavior of Volatility, Serial Correlation and Investment Horizon, "*Q-Group,* Fall Proceedings, 2014; A. Ang, *Asset Management: A Systematic Approach to Factor Investing,"* Oxford University Press, 2014; and G. Gorton, and G. K. Rouwenhorst, "Facts and Fantasies about Commodity Futures," *Financial Analysts Journal*, Vol. 62(2): 47–68, 2006.

26 The sample correlations would not be different if the return means were different from zero ($\mu \neq 0$). For different samples we find either of the correlations (using 3y or 6m returns) are higher or lower. As the sample size increases (say at 100k observations, *i.e.,* 500 centuries) the correlation derived using 3y or 6m returns converge to the true correlation for any sample.

Figure 10: 99% Confidence Interval of Estimated Correlation Values Based on Sample Size
(1m Returns)



Note: To determine the correlation 99% confidence interval and average we generate 10,000 samples of 120 and 600 observations, respectively, from the true joint distribution. The figure shows 99% confidence intervals for the correlation value across the 10,000 samples, depending on the sample size. The figure also shows the mathematically calculated 99% confidence interval for a sample size of 600 observations.
Source: PGIM IAS. For illustrative purposes only.

Suppose we decide to use non-overlapping 3y observations. Starting at the first 6m period in s_1, the 16 non-overlapping 3y returns produces yet another correlation value, this time 0.63. However, the investor could also form other sets of non-overlapping 3y returns starting anywhere between the second to sixth 6m period, generating five more 3y correlation estimates with a remarkably wide range [-0.20, 0.63; Figure 11]! However, the average of these six 3y correlations is close to the overlapping 3y returns correlation (0.14 vs. 0.12), suggesting that using overlapping observations removes sampling variability associated with a random start date.

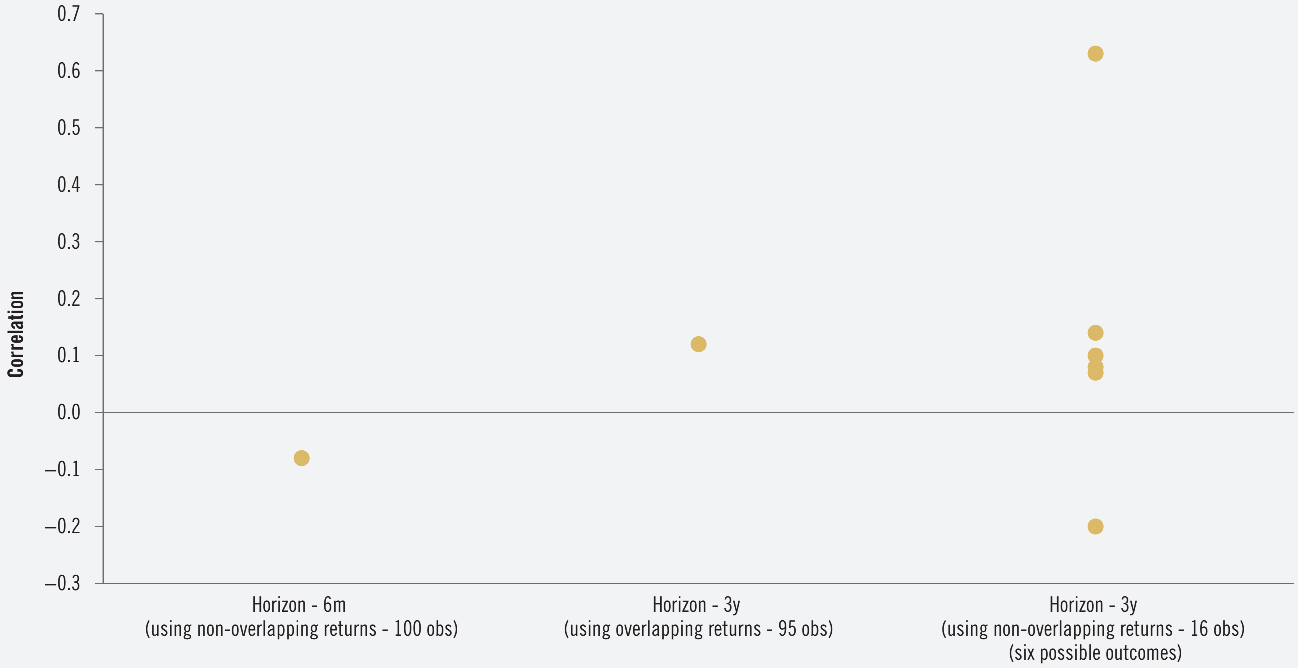
From the s_1 sample of 100 6m returns we have very different correlations depending on the return horizon: -0.08 using 6m returns, 0.12 using 3y overlapping returns and anywhere between -0.20 and 0.63 using non-overlapping returns (with an average of 0.14).

So far, we have assumed that the true 6m and 3y return horizon correlations are the same (-0.1). Even so, estimated correlations can differ significantly depending on the assumed return horizon, even with the same data. No wonder the literature has generated conflicting conclusions. Furthermore, the *true* correlation value may vary depending on the return horizon. Short horizon returns may be highly sensitive to short-term factors such as momentum and the investor's risk aversion. While these short-term factors may cancel out over time, factors such as real production costs and availability of substitutes may have an influence on longer horizon returns. Unfortunately, for investors there is no way to be sure whether the observed differences in short and long-term return horizon correlation estimates are due to sampling error, a difference in the true underlying return generating process, or both. Nevertheless, portfolio construction can try to account for the uncertainty in correlation estimation.

Alternative Approaches for Measuring Correlation Estimation Error

Although investors rarely have the opportunity to draw more data samples we can examine how the correlation estimate could vary. With 10,000 samples of 100 6m returns drawn from the assumed known joint distribution the average correlation value is -0.1 regardless of the length of return horizon (6m or 3y), or whether the returns are overlapping or not. So, there is no bias depending on the assumed return horizon. However, when 3y horizon returns (non-overlapping) are used the 99% confidence interval for the correlation value is [-0.69, 0.56], and when 3y horizon returns (overlapping) are used it is [-0.56, 0.42], compared to a 99% confidence interval of [-0.35, 0.16] when 6m returns are used.

Figure 11: Estimated Correlation Values Based on Return Horizon
(6m or 3y)



Note: Figure shows estimated correlation values for different choices – 6m or 3y (overlapping or non-overlapping) return horizon. For 3y and non-overlapping choice there are six possible starting periods and therefore six possible correlation estimates.
Source: PGIM IAS. For illustrative purposes only.

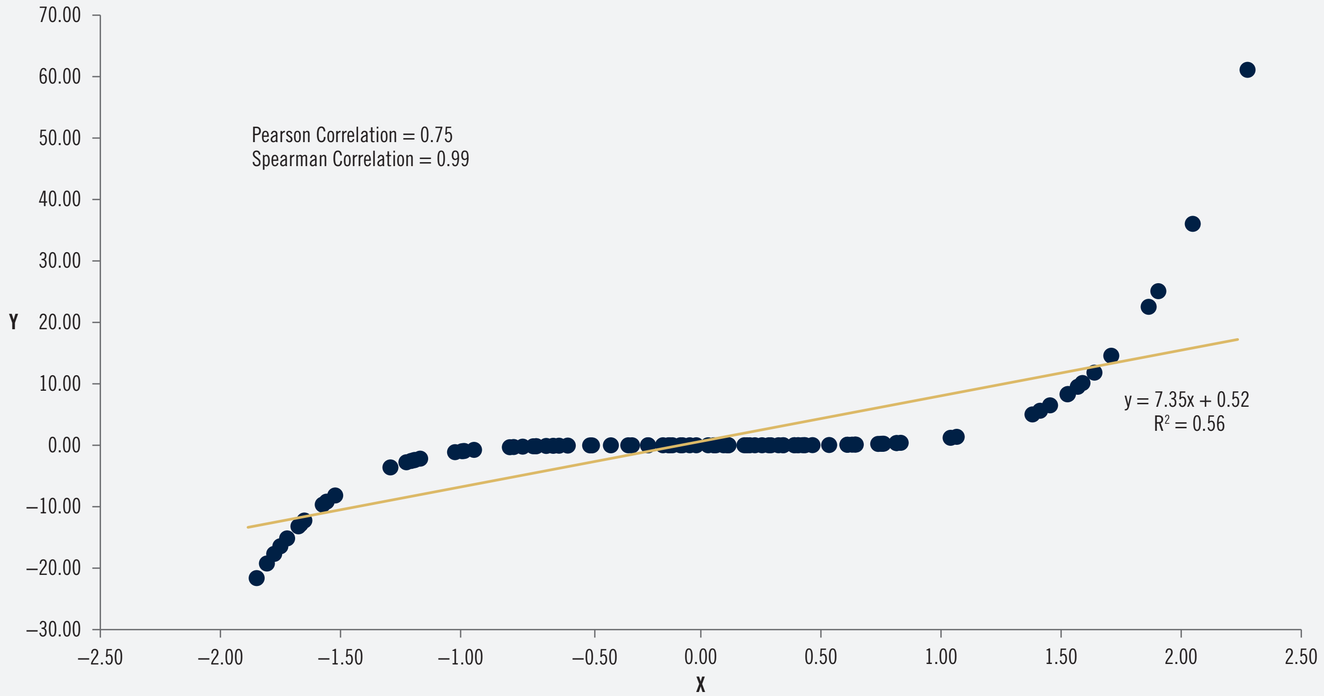
In reality, we do not know the true parameter value or the underlying distribution, we only have a finite set of data. We saw how widely the estimated correlation value could differ from the true value especially for small samples and/or long horizons. For some of the asset classes considered here we only have 50y of monthly returns data (or 100 6m returns or 10 5y returns). In such a situation it is challenging to estimate not only the true correlation parameter for a given return horizon but also the range of possible correlation values (akin to measuring a standard error).

In this situation we can resort to a "bootstrapping" approach first used by Efron (1979) that does not make any underlying distributional assumption.[27] In a bootstrap, we generate a "new" data sample (or, time series) by re-sampling *(with replacement) n* times from the original sample (*n* being the number of observations in the original sample). We then calculate the mean (or any other statistic) of the bootstrapped sample. The process is repeated *B* times (*i.e.*, *B* is the number of generated bootstrapped time series) resulting in *B* values for the statistic of interest. (In case of multiple time series the observations are re-sampled jointly to preserve any underlying correlation structure.) We can use these *B* values to estimate standard errors or confidence intervals. So, even though we only had one original time series, which may have relatively few observations, we can estimate the magnitude of the sampling error.

We illustrate the statistical properties of this bootstrapping approach using both overlapping and non-overlapping returns from the *single random sample*, s_1, of 100 6m returns for two random time series with a true correlation value of -0.1. First, we estimate confidence intervals for the true correlation value using overlapping data by randomly selecting, with replacement, 3y returns from the 95 3y returns (at 6m intervals and *overlapping*). We select 16 of such 3y returns (*i.e.*, 48y) and compute a correlation. We repeat the process 10,000 times using the same sample s_1. From these 10,000 correlation values we calculate an average correlation of 0.11 and a 99% confidence interval for the correlation value for 3y horizon returns of [-0.48, 0.63].

27 B. Efron, "Bootstrap Methods: Another Look at the Jackknife," *The Annals of Statistics,* vol. 7 (1), 1979.

Figure 12: Spearman vs. Pearson Correlation



Source: PGIM IAS. For illustrative purposes only.

If we use the bootstrap approach and *non-overlapping* 3y returns from s_1 then the average and the 99% confidence interval for the correlation values will depend on the starting 6m period when generating 3y non-overlapping returns. For example, using the first 6m starting period produces an average correlation value of 0.62 and 99% confidence interval of [0.00, 0.84] whereas using the third starting period produces an average correlation value of 0.10 and 99% confidence interval of [-0.45, 0.84].[28] Although the true correlation is -0.1, the wide range of the confidence interval suggests the investor may estimate either negative or positive values from the available sample data.

A requirement to use bootstrap methods is that the data sample contains observations that are independent and identically distributed (i.i.d.). For time series data, especially macroeconomic variable data, this condition is not usually satisfied, although asset returns in a competitive market are usually i.i.d. The challenge to use bootstrap techniques with the time series data is to generate an i.i.d. data sample that preserves any dependency structure across observations. For the estimated 6m correlations in Section 2 we employ a "moving block" bootstrap and define the data sample to be 6m blocks of consecutive monthly data. To construct a bootstrap sample, we resample the 6m blocks – a random reshuffling of the 6m blocks, with replacement. Defining the data in terms of blocks tries to preserve any dependency structure within a block while the random choosing of blocks tries to maintain independence across blocks.

To summarize, correlation estimates can vary widely based on the data sample size and length of the return horizon. Since the estimated correlation value may be far from the true value an investor should select the correlation value appropriate for their return horizon and then estimate the sampling error of the correlation value using a bootstrap approach.

28 For this example, the data are at 6-month intervals, so there can be 6 different starting periods for 3y returns, *i.e.,* 6 different time series of 3y non-overlapping returns. We can conduct non-overlapping bootstrap simulation (10,000 times) separately for each of the 6 different starting periods. Using all 60,000 correlations the average correlation value is 0.13 with a 99% confidence interval of [-0.58, 0.85].

Pearson vs. Spearman Correlation

For the examples in this section we have been using the Pearson correlation which is a measure of linear correlation between two variables (*i.e.*, how closely do the observations fit along a line?). However, for the correlation estimates reported in Section 2 we used Spearman correlation which measures the (monotonically) increasing or decreasing relationship, not necessarily linear, between two variables. To estimate Spearman correlation the values of each of the two variables are ranked (*e.g.*, largest down to smallest) and then the correlation is estimated using the rank values. Figure 12 shows two variables, x and y, that can have different Pearson and Spearman correlation values. The Pearson value is 0.75 but (as expected) the Spearman value is higher at 0.99.

An advantage of the Spearman correlation is that it is less sensitive to outliers in the tail because extreme values are limited by their rank value. The Pearson correlation is more sensitive to outliers. Outliers may be more common for shorter horizon returns.[29] So, in order to compare correlations for different investment horizons we use Spearman correlations.

Acknowledgments

We wish to thank Drs. Taimur Hyat and George Sakoulis for their valuable comments and suggestions.

29 See J. Fan and Q. Yao, *The Elements of Financial Econometrics,* Cambridge University Press, 2015.

APPENDIX

Figure A1: Asset Class Sensitivities to Macroeconomic and Market Variables: -1σ and +1σ Percentile Betas
(USD Total Returns,; January 1973 – January 2019)

		-1σ Percentile Beta					+1σ Percentile Beta				
		6m	1y	2y	3y	5y	6m	1y	2y	3y	5y
Gold	Constant	–0.12	–0.23	–0.18	–0.15	–0.16	–0.03	–0.05	0.01	0.06	0.12
	DXY	–1.17	–1.41	–1.65	–1.76	–1.95	–0.69	–0.71	–0.86	–0.78	–0.49
	|ΔVIX|	0.75	1.15	–0.21	–2.37	–4.64	3.55	7.12	7.09	6.56	6.65
	Δeff_FF	–0.06	0.08	0.33	0.34	0.09	0.18	0.80	1.03	1.16	1.50
	CPI	1.40	1.40	0.26	0.03	–0.23	4.73	5.06	3.55	3.12	3.27
	CFNAI	–0.04	–0.14	–0.17	–0.22	–0.32	0.02	0.01	–0.02	–0.04	–0.07
Gold Miner Equity	Constant	–0.05	–0.15	–0.15	–0.15	–0.17	0.08	0.14	0.16	0.20	0.27
	DXY	–1.23	–1.62	–2.15	–2.67	–3.50	–0.59	–0.50	–0.88	–1.01	–0.89
	|ΔVIX|	–2.09	–3.98	–5.57	–8.54	–13.19	2.76	6.10	5.34	4.34	3.86
	Δeff_FF	–0.21	–0.54	–0.35	–0.55	–1.48	0.13	0.39	0.70	0.70	0.79
	CPI	–0.21	–0.09	–0.07	0.40	–0.48	3.63	4.70	3.91	4.25	6.53
	CFNAI	–0.05	–0.13	–0.19	–0.23	–0.28	0.04	0.10	0.06	0.04	0.10
US Equity	Constant	0.11	0.21	0.17	0.13	0.05	0.16	0.33	0.30	0.28	0.30
	DXY	–0.18	–0.25	–0.26	–0.04	–0.12	0.17	0.27	0.48	0.93	1.35
	|ΔVIX|	–3.63	–7.34	–6.66	–6.24	–7.79	–1.75	–3.10	–1.94	0.30	3.61
	Δeff_FF	–0.18	–0.47	–0.47	–0.45	–0.70	–0.02	0.01	0.09	0.23	0.76
	CPI	–1.44	–1.42	–1.26	–1.47	–2.23	0.30	0.55	0.82	0.84	1.66
	CFNAI	–0.01	–0.01	0.02	0.03	0.00	0.04	0.10	0.15	0.19	0.29
Commodity	Constant	–0.03	–0.08	–0.10	–0.16	–0.19	0.03	0.06	0.08	0.08	0.12
	DXY	–0.87	–1.04	–1.39	–1.69	–2.27	–0.42	–0.35	–0.37	–0.45	–0.54
	|ΔVIX|	–1.99	–3.22	–4.43	–4.50	–6.63	–0.27	1.28	2.44	5.12	6.34
	Δeff_FF	–0.06	–0.19	–0.25	–0.42	–1.35	0.11	0.34	0.43	0.45	0.30
	CPI	2.76	2.11	1.38	0.88	0.28	5.13	4.39	3.97	4.00	4.85
	CFNAI	0.00	–0.02	–0.06	–0.07	–0.07	0.05	0.11	0.11	0.14	0.26
Treasury	Constant	0.02	0.03	0.02	0.00	–0.03	0.03	0.07	0.06	0.06	0.07
	DXY	–0.09	–0.10	–0.15	–0.20	–0.25	0.04	0.14	0.12	0.12	0.21
	|ΔVIX|	0.08	–0.45	–0.56	–0.56	–1.25	0.58	0.84	1.05	1.48	2.35
	Δeff_FF	–0.10	–0.33	–0.47	–0.53	–0.71	–0.06	–0.18	–0.25	–0.25	–0.23
	CPI	–0.11	0.10	0.22	0.18	0.01	0.34	0.65	0.95	1.16	1.55
	CFNAI	0.00	0.00	0.02	0.02	0.01	0.01	0.03	0.06	0.07	0.10
TIPS	Constant	0.01	0.01	0.00	–0.01	–0.02	0.02	0.05	0.05	0.04	0.04
	DXY	–0.21	–0.20	–0.27	–0.32	–0.40	–0.09	–0.02	–0.07	–0.08	–0.05
	|ΔVIX|	–0.24	–0.67	–0.60	–0.27	–0.37	0.42	0.86	1.02	1.46	1.96
	Δeff_FF	–0.11	–0.28	–0.36	–0.39	–0.54	–0.06	–0.15	–0.22	–0.22	–0.21
	CPI	0.59	0.60	0.70	0.71	0.54	1.18	1.22	1.24	1.30	1.34
	CFNAI	–0.01	–0.02	–0.01	–0.01	–0.01	0.01	0.02	0.03	0.03	0.05

Source: PGIM IAS. For illustrative purposes only.

Figure A2: Index Mapping

Gold	Gold Bullion LBM $/t oz	Datastream
Gold Miner Equity	Americas-Datastream Gold Miner Index	Datastream
Gold Future	S&P GSCI Gold Index	Datastream
US Equity	US Datastream Market Index	Datastream
US Energy Equity	US Datastream Oil & Gas Index	Datastream
Commodity (GSCI)	S&P GSCI Index	Datastream
Treasury	Bloomberg Barclays Treasury Index	Datastream
US Corporate BAA	Bloomberg Barclays Baa-only US Corporate Index	Datastream
TIPS	Pond and Mirani, Barclays (January 1973 – January 1997) and Bloomberg Barclays TIPS Index (February 1997 – January 2019)	Barclays, Datastream
Cash	3m Treasury Bill: Secondary Market Rate	St. Louis Fed
CPI	Consumer Price Index for All Urban Consumers: All Items, Index 1982-1984=100, Monthly, Seasonally Adjusted	St. Louis Fed
CFNAI	Chicago Fed National Activity Index, Index, Monthly, Not Seasonally Adjusted	St. Louis Fed
Effective Fed Funds Rate	Effective Federal Funds Rate, Percent, Monthly, Not Seasonally Adjusted	St. Louis Fed
VIX	CBOE S&P 100 Volatility Index: VXO, Index, Monthly, Not Seasonally Adjusted	St. Louis Fed, PGIM IAS
US Dollar	US Dollar Index DXY	Datastream

Source: PGIM IAS.

We split the sample period into two: February 1968 – January 1992 (because of the split, we go back further in history to 1968 but with fewer assets), a period with higher inflation expectations and supply-led inflation (OPEC oil embargo and Gulf War I); and February 1992 – January 2019, a period of easy monetary policy and tame inflation expectations. See Figure A3.

Figure A3: Sub-sample Regressions; Average Betas
(USD Total Returns; February 1968 – January 2019)

		1968–1991					1992–2019				
		3m	6m	12m	24m	36m	3m	6m	12m	24m	36m
Gold	Constant	**–0.07**	**–0.16**	**–0.41**	**–0.43**	**–0.66**	0.00	–0.04	**–0.17**	**–0.26**	**–0.30**
	$\lvert\Delta\text{VIX}\rvert$	**1.07**	2.29	6.04	7.66	**12.01**	0.17	0.69	2.68	4.61	5.03
	Δeff_FF	**–0.05**	–0.05	–0.03	0.23	0.22	0.02	0.09	0.41	**0.90**	**1.29**
	CPI	**4.86**	**5.16**	**6.04**	**5.17**	**6.55**	2.18	**4.09**	**7.16**	**8.90**	**9.74**
	CFNAI	**0.02**	**0.06**	0.16	**0.23**	**0.34**	–0.01	–0.04	**–0.13**	**–0.20**	**–0.26**
US Equity	Constant	**0.10**	**0.18**	**0.34**	**0.31**	**0.41**	**0.06**	**0.15**	**0.33**	**0.27**	**0.23**
	$\lvert\Delta\text{VIX}\rvert$	**–0.90**	**–1.94**	**–3.65**	–4.30	**–5.49**	**–1.42**	**–3.35**	**–6.17**	–2.68	–0.09
	Δeff_FF	–0.03	–0.08	–0.19	–0.22	–0.01	–0.04	–0.02	–0.11	0.07	0.31
	CPI	**–2.75**	**–2.34**	–1.95	–1.48	**–2.36**	–0.57	–1.29	–2.77	–3.52	–4.64
	CFNAI	–0.01	–0.01	–0.02	–0.01	–0.05	0.02	0.02	0.10	**0.16**	**0.19**
Treasury	Constant	**0.03**	**0.06**	**0.14**	**0.16**	**0.18**	**0.01**	0.02	0.02	0.00	0.02
	$\lvert\Delta\text{VIX}\rvert$	0.08	0.02	–0.25	–1.21	–1.59	0.17	0.43	0.41	0.67	–0.24
	Δeff_FF	**–0.02**	**–0.06**	**–0.15**	**–0.19**	**–0.17**	–0.02	**–0.08**	**–0.22**	**–0.34**	**–0.40**
	CPI	**–0.76**	**–0.68**	–0.71	–0.58	**–0.79**	–0.52	–0.05	0.85	**1.45**	**1.74**
	CFNAI	**–0.01**	–0.01	–0.01	–0.01	–0.01	0.00	0.01	0.01	**0.04**	**0.03**
Cash	Constant	**0.01**	**0.03**	**0.07**	**0.08**	**0.09**	**0.01**	**0.01**	0.01	0.00	0.00
	$\lvert\Delta\text{VIX}\rvert$	0.01	0.00	–0.03	–0.09	–0.35	0.00	0.01	–0.07	–0.19	–0.73
	Δeff_FF	**0.00**	–0.01	–0.01	0.00	–0.01	**–0.01**	–0.02	–0.07	**–0.14**	**–0.26**
	CPI	**0.27**	**0.28**	0.19	0.07	–0.05	**0.20**	**0.40**	**0.83**	**1.47**	**1.84**
	CFNAI	**0.00**	**–0.01**	**–0.02**	**–0.02**	**–0.03**	**0.00**	**0.01**	0.02	**0.03**	**0.03**

Source: PGIM IAS. For illustrative purposes only.

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