THE TRADEOFF BETWEEN LIQUIDITY AND PERFORMANCE

Private Assets in Institutional Portfolios

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The PGIM Institutional Advisory & Solutions group advises institutional clients on a variety of asset allocation and portfolio construction topics, and delivers bespoke research based on an institution's specific objectives.

Institutional investors have increased their allocation to private assets to capture a potential return premium over public assets and diversification benefits. However, an allocation to private assets, which typically are less liquid than public assets, raises questions: "How does an allocation to private assets affect my portfolio's ability to satisfy my cash liabilities?"; "What is the marginal cost, in terms of portfolio returns, to increase my confidence of always having enough liquidity to satisfy these liabilities?"; and "What is my optimal allocation to private assets and the composition of the private and public portfolios?".

"Liquidity" for investors is their degree of confidence that their portfolios will satisfy cash flow obligations. Investors who believe their portfolios are liquid are highly confident that they can meet all cash flow obligations across different economic scenarios – even if the portfolio may contain some less-liquid private assets. In contrast, investors who state that their portfolios are not very liquid have less confidence they will always be able to meet all their obligations.

We propose a public-private asset allocation framework that incorporates the characteristics of common illiquid private assets (limited partnerships, or LPs). We assume the investor tries to maximize expected horizon portfolio value provided that the investor's future cash obligations are all met with a pre-specified minimum degree of confidence. Risk to the investor is not short-term volatility but failing to meet cash obligations.

The framework highlights that the optimal public-private asset allocation, and the optimal asset allocation within both the public and private portfolios, are interrelated. In addition, the model shows that increasing a portfolio's liquidity – *i.e.*, increasing the confidence of meeting all cash obligations – implies a less risky portfolio, with

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a lower allocation to less liquid, private assets. However, this may be expensive in terms of the expected future horizon value of the portfolio. The investor faces a fundamental tradeoff: liquidity *vs*. performance. If an investor chooses to be more confident of meeting all future cash obligations, how much will this additional liquidity cost in terms of expected future portfolio value? Some investors may conclude that their portfolios may be too liquid.

Introduction

Institutional investors have made substantial allocations to private markets (*e.g.*, equity, credit, and real estate) with the expectation of capturing a private market premium over public market counterparts and gaining a diversification benefit.[1]

While private assets may help improve portfolio performance, their risk and return characteristics, as well as their limited liquidity compared to traditional public assets, raise issues for a chief investment officer:

- How does my allocation to private assets affect the probability that I will satisfy my cash liabilities?
- How much does it cost to increase my confidence of having enough liquidity to meet my liabilities?
- What is my optimal allocation to private assets?
- How does my private asset allocation affect the composition of my public portfolio, and *vice versa?*

How can a chief investment officer approach answering these questions? Using a traditional, single-period asset allocation framework (*e.g.*, mean-variance optimization) typically begins by recognizing that private asset periodic valuations do not reflect transactable values and are too smooth over time. Private asset returns are then "unsmoothed", in some fashion, to make them fit within the framework along with public assets. The investor then maximizes expected return subject to a certain level of short-term volatility risk. However, as argued in Shen and Phelps (2018), this traditional allocation framework, with its focus on short-term volatility risk, fundamentally assumes the assets are all tradeable and portfolio holdings can be rebalanced, which is not realistic with many types of private assets. Although in some market environments private assets may be transacted in a secondary market as needed, they have relatively high transaction costs or may not be tradeable during challenging markets.

Instead of maximizing returns subject to short-term volatility risk, Shen and Phelps (2018) approach the asset allocation problem by arguing that an investor tries to maximize the horizon value of their portfolio subject to having sufficient liquidity to always meet cash obligations with a certain degree of confidence. In the context of an investment universe containing assets with widely varying liquidation characteristics, the meaning of portfolio "liquidity" for the investor is the degree of confidence in meeting cash obligations. Investors who claim their portfolios are highly liquid mean that they are highly confident that they will meet all their cash obligations, across different economic scenarios – even if the portfolio may contain illiquid, private assets. In contrast, investors who state that their portfolios are not very liquid mean that they have relatively low confidence they will meet all their obligations across different economic scenarios.

Increasing a portfolio's liquidity – *i.e.*, increasing the confidence of meeting cash obligations over the horizon – generally implies that the portfolio must become less risky and have a lower allocation to less liquid, private assets. However, this may be expensive in terms of the expected future horizon value of the portfolio. The investor faces a fundamental tradeoff: liquidity *vs*. performance. If an investor chooses not to be 100% confident of meeting all future cash obligations, how much might expected performance improve?

To answer this question, we propose a simulation-based asset allocation framework that assumes investors maximize their expected horizon value subject to meeting their interim cash obligations over the investment horizon, with a specified level of confidence (*i.e.*, their "liquidity requirement"). This framework incorporates realistic performance characteristics of common private assets with limited liquidity (*i.e.*, limited partnerships, or LPs) which is not found in traditional mean-variance asset allocation frameworks. Specifically, we build an empirically estimated relationship between LP performance and public market performance, incorporating some unique characteristics of LP investments: delays until the first capital call and an empirical call schedule of capital allocated to LP investments. The framework also allows investors to express views on how LP asset class performance (versus public markets) might differ from historical experience. Furthermore, recognizing that an LP investor's fund-selection skill is key in private asset performance, the framework allows investors to examine how their optimal asset allocation might vary depending on their fund-selection skill.

The framework highlights that the asset allocation between private and public assets, as well as the asset allocation within the private and public portfolios themselves, are all interrelated. There are many ways to satisfy a liquidity requirement: less allocation to private assets; more allocation to private assets but a less risky public portfolio; a less risky private portfolio but a higher risk

1 See J. Shen and B. Phelps, "Illiquid Private Assets: Interaction of Illiquid and Liquid Assets in Investor Portfolios", PGIM, February 2018, for specific systematic risk factors that comprise a "private market premium".

public portfolio; etc. The proposed framework can help investors determine their optimal allocation across both private and public assets to achieve their liquidity goals. The framework also helps investors measure the sensitivity of the optimal allocation to changes in assumptions about the risk and return characteristics of both private and public assets (*i.e.*, "what-if analysis").

Importantly, the framework allows investors to answer, "How much does it cost to make my portfolio more liquid?". Investors may find that portfolio liquidity comes at a high cost. Is this cost worthwhile? Being able to measure the cost of liquidity helps them make a more informed business decision that is suited for their circumstances. "Maybe my portfolio is too liquid?"

The framework is flexible and highly customizable. Most of the components in the framework may be adjusted, individually or collectively, to accommodate unique investment objectives and constraints corresponding to different types of investors. For example, following the trend of transitioning from Defined Benefit (DB) plans to Defined Contribution (DC) plans, some corporate pension plans are closed. These pension portfolios usually have a heavy and increasing percentage of retirees with almost certain cash obligations. Therefore, liabilities are in "run-off" mode, and the portfolio is often referred to as an "end-state" portfolio. Pure immunization with fixed income assets may not be the best strategy to manage such "end-state" portfolios. One possible explanation is that there is mortality risk that could cause the actual cash liabilities to deviate from the estimated ones. For example, if on average, participants live longer than expected, the pension portfolio could be exposed to the risk of having to pay more than currently expected. This risk may argue for the inclusion of return-seeking assets such as private assets. A CIO can use the asset allocation framework to help solve for the optimal allocation of public and private assets for an "end-state" portfolio. A key adjustment is typically to add a constraint to minimize interim funded status volatility over the horizon.

We present the asset allocation framework using a Case Study. We make several baseline assumptions, but also highlight the framework's flexibility to accommodate an investor's own assumptions or views. Following the Case Study, we present the framework's modeling details.

Case Study

The Case Study illustrates the application of the framework. We first frame the investor's asset allocation problem, including the investment objective and constraints. We then introduce the investor's investment opportunity set as well as how we model each asset's future performance.

The Asset Allocation Results section shows the optimal asset allocation produced by the framework. This section highlights the interrelationship between the investor's liquidity preference and portfolio performance, which allows the investor to deduce their own "cost of liquidity". In addition, we analyze the impact of changing views of assets' performance and private asset transaction costs on the optimal asset allocation. Finally, given that our asset allocation framework is simulation-based, we present the distribution of possible portfolio horizon values, identifying those situations in which the investor's liquidity requirement failed. For those failed simulation runs, we show the distribution of the time until the liquidity failure, *i.e.*, the first month across the investment horizon in which there is insufficient cash to meet a liability payment.

The Investor's Asset Allocation Problem

We assume that risk to the investor is not short-term portfolio volatility but that the portfolio fails to meet a cash liability at any given time during the investment horizon. Consequently, we assume that the investor maximizes 10y horizon expected portfolio value subject to the constraint that the portfolio will satisfy future cash obligations with a certain minimum likelihood. For example, an investor may seek to maximize their portfolio's horizon value provided that, with at least 90% likelihood, the portfolio satisfies the cash liabilities in *all months* over the horizon.

In other words:

Maximize $\quad \textit{Expected MV Portfolio}_{Horizon}$

$$\text{s.t., Pr}\{(\textit{MV Portfolio}_t > \textit{Cash Liability}_t) \textit{ for all t from 1 to T}\} \geq \textit{Confidence Level (\%)}$$

For an investor who must decide how to allocate across assets, given a confidence level (*i.e.*, the investor's liquidity requirement), the optimal allocation result not only provides the allocation between public and private assets, but also the allocation within both the public and private portfolios.

To generate an asset allocation solution, investors need to provide the following key inputs:

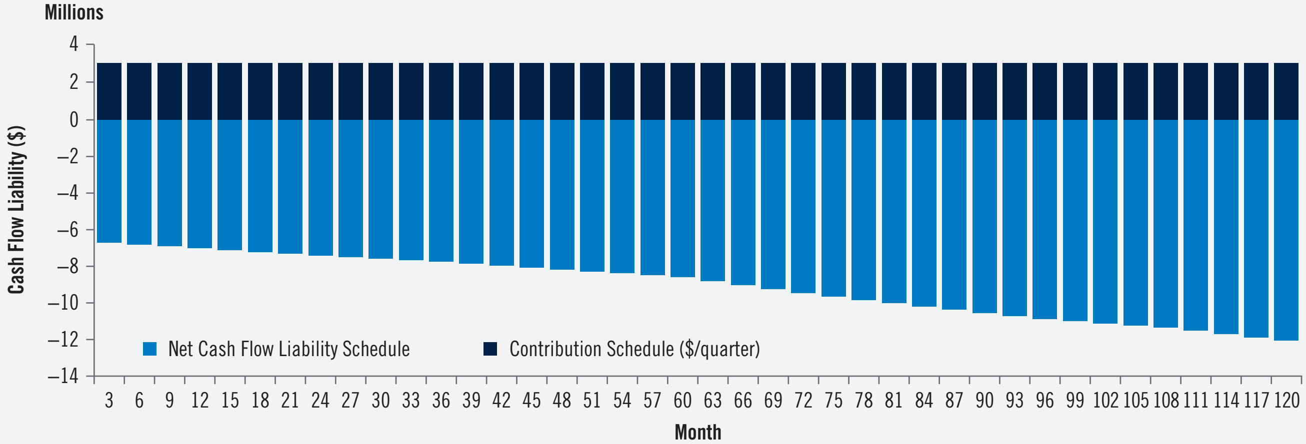
- Expectations regarding public assets' market performance;
- Investor's private asset performance views, if any, either at the asset class or fund level; and
- Cash flow liability schedule (including future contributions, if any) for the investment horizon period.

The Investor's Asset Allocation Constraints

Key Constraint: Investor's Liquidity Requirement

The framework allows investors to specify their investment constraints. In this Case Study, the (only) constraint is that the portfolio needs to satisfy a series of periodic (*e.g.*, monthly or quarterly) cash obligations with a certain minimum likelihood. At the time of a cash obligation, assets are sold from the portfolio and the proceeds (after transaction costs) are used to satisfy (if possible) the cash requirement. The framework also allows investors to specify future cash contributions to the portfolio. We propose a hypothetical quarterly cash flow liability schedule alongside a quarterly contribution schedule to capture our hypothetical investor's liquidity requirement (Figure 1). The Case Study assumes that cash outflows rise gradually over time and that at each quarter-end the portfolio receives a $3m cash inflow. The net effect is a rising net cash flow liability schedule.

Figure 1: Net Cash Flow Liability Schedule
(hypothetical)



Note: The figure shows the assumed increasing quarterly net liability cash flow obligations and constant quarterly cash contributions for 120 months.

Source: PGIM IAS. Example shown for illustrative purposes only.

The asset allocation framework assumes all assets, including illiquid private assets, can be sold prior to the horizon to satisfy the liquidity requirement. LP assets typically have higher transaction costs than public assets. The assumed private and public asset transaction costs for the Case Study are shown in Figure 2.[2] Users of the framework are free to specify their own expected transaction costs. It is a useful exercise to examine how the optimal asset allocation results might change under different transaction cost assumptions. The framework tries to reflect that during bad economic times (*e.g.*, environments of poor financial market performance) it is typically more expensive to liquidate LP investments. Consequently, the framework has different transaction costs depending on whether LP sales occur during "good" or "bad" states of the economy.

Figure 2: Assumed Transaction Costs of Public Assets & LP Investments
(as % of Allocation Value)

Transaction costs (%)	Public (IG) Debt	Public Equity	LP Buyout	LP Mezzanine Debt	LP Real Estate
"good" economy	0.6%	0.1%	5.0%	5.0%	5.0%
"bad" economy	1.0%	0.1%	30.0%	15.0%	9.0%

Note: These are the assumed transaction costs for the Case Study. A "bad" state of the economy is defined as when the monthly moving average (6m, backward-looking) of public equity total returns experiences a drawdown of more than -15%. Our choice of a 6m lookback period captures prolonged and sustained periods of downturns. The framework could accommodate many other possible definitions of bad and good economies. The framework does not consider price impact costs.

Source: PGIM IAS. Provided for illustrative purposes only.

2 Some information on LP buyout secondary market transaction costs is contained in: T. Nadauld, B. Sensoy, K. Vorkink, and M. Weisbach, "The Liquidity Cost of Private Equity Investments: Evidence from Secondary Market Transactions," NBER, July 2016.

In recognition of the higher transaction costs associated with the sale of LP investments, we specify a "selling rule" to guide the simulation on which of the five assets to sell, and in which order, to generate cash. The details of the selling rule can be found in Appendix A2. Other selling rules can be considered.

Other Constraints

In addition to the liquidity requirement constraint, investors may specify other investment constraints as desired. For example, it may be important to keep a pension portfolio's funded status volatility low, especially when it is approaching its "end-state". Therefore, pension fund managers would likely prefer to add a maximum funded status volatility constraint.

Investment constraints can also be added to address unique investment concerns. For example, private pension plans contemplating a future transfer of their pension risks, either through offering lump-sums to participants or executing a Pension Risk Transfer (PRT) transaction, may try to keep their allocation to private assets under a certain threshold to avoid potentially large haircuts for liquidating or transferring private assets. Another example of an investment constraint might be related to regulatory requirements, such as a solvency constraint.

Asset Choices and Dynamics

Investment Opportunity Set

We assume there are five assets available in the investment opportunity set: two public assets and three private assets. The two public assets include a public "low-risk" asset and a public "high-risk" asset. Public assets are infinitely divisible (*i.e.*, small amounts can be sold), and transactions may occur at any time, at relatively low cost. Investors can specify their expectation of public asset performance over the 10y investment horizon. An example is shown in Figure 3. The framework simulates the dynamics of public asset returns from a joint t-distribution based on the specified expected returns, standard deviations, and long-term historical correlation. In the Case Study, the public low-risk asset is assumed to have the same monthly total return distribution as the Bloomberg Barclays 5-10y US IG Corporate Index, and the public high-risk asset matches the monthly total return distribution of the S&P 500 Index. We assume the correlation between the two liquid assets equals the correlation between the respective indices from October 1988 to December 2017.[3]

The three private assets are: LP buyout private equity, LP mezzanine debt and LP real estate.[4] These assets are "LP investments", as institutional investors typically gain significant exposure to private assets *via* a limited partnership (LP) vehicle. LP investments are considered less liquid than public assets: they are not infinitely divisible (*i.e.*, transactions are lumpy), their secondary market transactions are relatively infrequent, and they have higher transaction costs compared to public assets. The investor can determine whether to include any or all of these private assets in their investment opportunity set.[5]

Some LP investments (*e.g.,* LP mezzanine debt) generate periodic income shortly after the investment is made, which is an attractive feature for investors. For LP investments that generate interim (*e.g.*, quarterly) income, the assumed cash flow is expressed as an annual percentage rate as specified by the investor. For example, an investor may state that their LP mezzanine investment will generate 6% annually (as a percentage of their LP mezzanine allocation), payable quarterly. This income can be used to meet cash flow obligations (incurring no transaction costs). Any leftover income is invested *pro rata* into public assets. If, over the horizon, any of the LP mezzanine allocation is sold to meet liabilities, then the income generated will be reduced *pari passu*. As discussed below, the future value of these cash flows is subtracted from the future horizon value of the LP mezzanine allocation.

Figure 3: Public Asset Expected Annual Total Return and Volatility Assumptions

	Public Low-Risk: 5-10y IG Corporates	Public High-Risk: S&P 500
annual total return	3.6%	8.0%
annual volatility	5.3%	14.1%

Note: These are the expectations of public asset performance assumed for the Case Study. Investors can make their own assumptions.

Source: Barclays POINT, Datastream, PGIM IAS. Provided for illustrative purposes only.

3 For the Case Study we use USD data only. The framework could accommodate making the correlation dependent on the state of the economy.

4 Appendix A1 contains descriptions of the three LP investment types.

5 Other LP types, such as venture capital, opportunistic real estate, or European private equity buyout, can be added to the investment opportunity set, or replace the three current LP investment types.

Unlike for public assets, it is difficult to specify high frequency (*e.g.*, monthly or quarterly) returns and volatilities for private assets since investors do not control when, and to what extent, their private assets are invested. Furthermore, it is difficult to value assets that trade infrequently. Instead of relying on user-supplied dollar-weighted (*e.g.*, IRR) returns and standard deviations for private assets which are incompatible with the time-weighted returns of public assets, the model relies on the estimated periodic interim value relationship (measured in dollars) between an LP investment and public assets.[6] For example, a dollar invested in a public asset today, with dividends or coupons reinvested, has an estimated value at the end of the horizon. We estimate the empirical relationship between the horizon value of public assets and the horizon value of an LP investment, as well as at interim horizons up to the final 10y horizon.

Not surprisingly, the relationship between LP investment horizon values and public market horizon values is noisy. Given a horizon value of the public markets, we can estimate the expected horizon value of the LP investment. However, the actual LP horizon value will fall within a distribution around this expected horizon value, reflecting the uncertainty of the empirical relationship. This prediction error volatility (a.k.a., the standard error of the prediction response) measures the dispersion of the actual LP horizon value around its expected value. The framework incorporates this uncertainty when calculating LP horizon values. For example, in a simulation run the public market horizon value produces an expected LP horizon value. However, to produce a final LP horizon value for that simulation run, we add a draw from the prediction error distribution. A "positive" ("negative") draw from the distribution means that the LP horizon value for that simulation run will be greater than (less than) the expected LP horizon value conditional on public market performance.

Incorporating Investor Views

We use this prediction error distribution to allow investors to express views on how they expect LP investments to perform relative to public assets. For example, if an investor has no view on LP performance the simulation will rely on the estimated expected relationship between LP and public asset horizon values, plus a random draw from the prediction error distribution, to produce an LP investment horizon value. But, if an investor believes that, going forward, LP investments will likely experience better than the estimated (*i.e.*, historical) relative performance, then the simulation run will tend to draw prediction error values that lie above the expected value. The more optimistic (pessimistic) the investor's view, the greater (lower) the likelihood that the simulation will draw a value from the prediction error distribution above the expected value.

So far, we have considered LP investments at the "vintage" (or, pooled) level. In other words, an investor is assumed to make an LP allocation across all LP funds available this year (*i.e.*, this year's vintage). This can be viewed as a generic allocation to LPs which may be appropriate for large investors who make numerous LP investments. However, other investors may invest in only a handful of LP funds.

A characteristic of private investments is that the range of fund-level LP performance can be wide, producing larger prediction error volatility compared to what is estimated at the vintage-level. Consequently, the framework allows investors to select the number of LP funds, and each fund's estimated horizon performance will incorporate this higher fund-level prediction error volatility. The simulation will generate an LP horizon value for each fund independently, and then will combine their performance. This permits some fund diversification benefits which increases (up to a point) as the investor selects more and more LP fund investments. The asset allocation framework also allows investors to provide LP performance views at the fund level, in a manner like that at the vintage level. For investors with access to well-performing GPs, this ability to express confident views on LP fund-level performance can have a material impact on the asset allocation results.

The Case Study's baseline scenario assumes that the investor has "no view" on how LP investments will perform relative to the benchmark public investments. In other words, over the next 10y, LP investments will perform the same as they have historically versus public markets. In addition, we assume that the investor has average fund-selection skill for all LP investment types. Finally, we assume for each LP investment type, the investor diversifies over 5 LP funds.

LP Allocation Value

The asset allocation framework recognizes that the performance of an **LP investment** – as reported by the GP – may differ from that of an **LP allocation** – what the investor commits to at the time of asset allocation. This performance difference arises from the potential delayed and gradual call of capital by the GP. We assume that any uninvested LP allocation is invested in a "default investment". Investors can define the "default investment" which may differ depending on the LP investment type. For the Case Study, we assume that the "default investment" for all LP types is the public asset portfolio (*i.e.*, Bloomberg Barclays US 5-10y IG Corporate Index and S&P 500 Index).

The asset allocation framework makes adjustments to the estimated LP investment performance to better approximate what an investor might experience at the time they make their LP allocation. In the Case Study, we assume – roughly reflecting the

6 See Section "Asset Allocation Framework - Detail" for details.

current environment – that the "stub" period (*i.e.*, the time until the first capital call is made) is 2y for LP buyout funds, 0y for LP mezzanine funds, and 1y for LP real estate funds. In other words, for LP buyout funds the Case assumes it will be 2y from the time the LP investor makes the capital commitment (*i.e.*, allocation) until the first capital call, which will likely be only a portion of the LP investor's total commitment.

The "Asset Allocation Framework – Detail" section describes the modeling of the LP allocation.

Asset Allocation Results

Based on the Case Study assumptions (see Appendix A3) and an initial portfolio value of $350m, our asset allocation framework produces an optimal asset allocation (Figure 4) which maximizes expected horizon portfolio value, subject to different levels of confidence that the portfolio, at each month over the horizon, can meet all its cash flow obligations.[7]

As Figure 4 shows, the optimal asset allocation changes as the investor's liquidity requirement (*i.e.*, confidence level) changes. For example, at a 90% confidence level (of meeting all cash flows over the entire investment horizon, across different economic scenarios), the optimal asset allocation that maximizes expected horizon value is 6% public (IG) debt, 65% public equity, 26% LP buyout, 2% LP real estate, and 1% LP mezzanine debt – a 29% total allocation to private assets. If the investor's liquidity requirement increases to a 95% confidence level, then the allocation to private assets declines to 27%, with a 10 percentage point decrease in LP buyout, and a combined 8 percentage point increase in LP real estate and LP mezzanine debt. While the increase in allocation to public assets is modest, there is a notable change in the composition of public asset allocation: a large increase in public IG debt and a large decrease in public equity. Moving from a 90% to a 95% confidence level is a challenging increase in the investor's liquidity requirement, but the model endeavors to maintain portfolio performance by retaining as much of the less liquid, but potentially better performing, private assets while simultaneously de-risking the public portfolio.

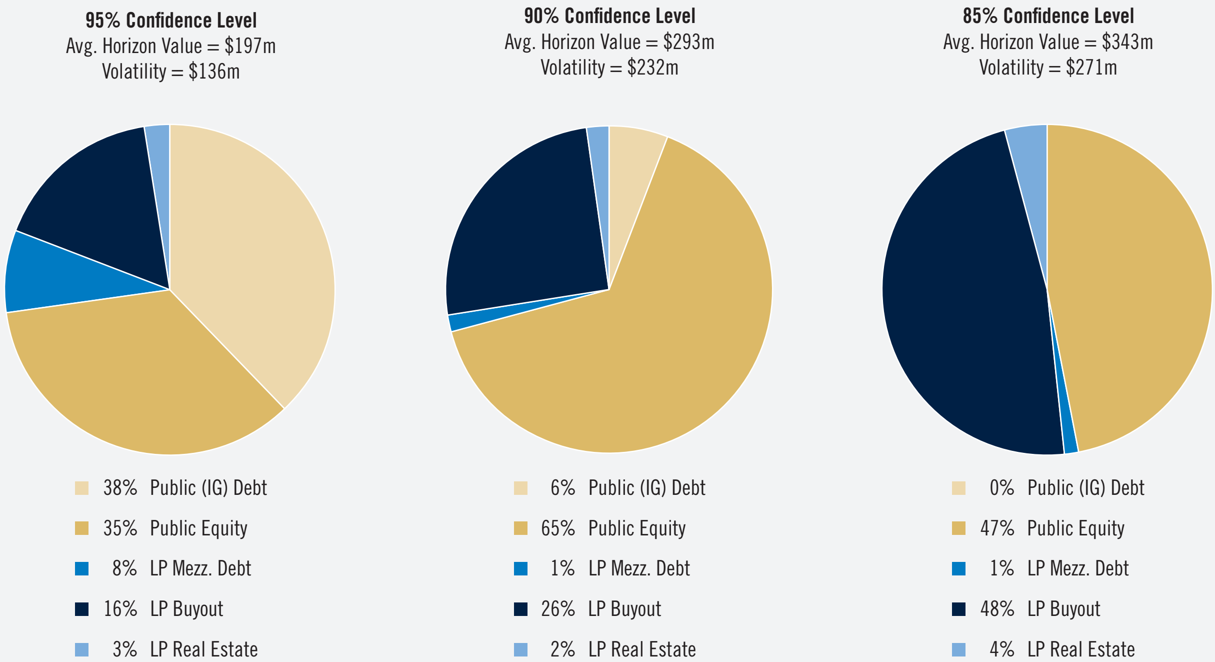
The attentive reader will note that moving from a 90% confidence level to a 95% confidence level causes a much larger reduction in expected horizon value (*i.e.*, a 33% reduction from $293m to $197m) compared to moving from an 85% confidence level to a 90% confidence level (*i.e.*, a 15% reduction from $343m to $293m). However, despite the same five percentage point change in the liquidity requirement, these changes reflect an increasing tightening of the liquidity requirement. Recall that the liquidity requirement is the investor's degree of confidence in always meeting cash flow obligations across all simulation runs. Therefore, for 5,000 runs, a 90% confidence level means that in 500 (or fewer) of the runs the portfolio comes up short of cash at some point during the horizon period. Increasing the confidence level to 95% limits the number of failed simulation runs to 250 (or fewer). A small 5% increase in confidence, starting from a 90% confidence level, requires a 50% decrease in the number of failures. In contrast, a 5% increase in confidence, starting from an 85% confidence level, requires the number of failures to drop by only 33%. As the liquidity requirement becomes elevated, further small increases in the liquidity requirement can become severely constraining. In fact, at very high liquidity requirement there may be no feasible asset allocation.

The sensitivity of the optimal asset allocation, and the expected horizon portfolio value, to changes in the liquidity requirement (*i.e.*, confidence level) depends on the opportunity set, the liability profile and the initial asset value. In general, at a low liquidity requirement (less than 90%), the optimal allocation and portfolio horizon value can move relatively smoothly as the required confidence level changes. However, when the liquidity requirement level is high, changes in the confidence level can produce much larger changes. At high liquidity levels, the portfolio struggles to meet all cash flow obligations over the entire horizon. Equity volatility becomes particularly problematic and the solution moves significantly towards public (IG) debt, with a large sacrifice in expected horizon value. Moving to a 95% liquidity requirement from a 90% level can produce a rather large change in asset allocation.

7 The optimal allocation is calculated by taking the average of the ten allocations with the highest horizon values. The allocations are evenly distributed around the specified confidence level. We include ten allocation candidates to avoid overlooking allocations that just fall short of the liquidity requirement caused by the inherent randomness involved in a simulation. See Section "Asset Allocation Framework - Detail" for details.

Figure 4: Optimal Asset Allocation
(Confidence Levels of 95%, 90% and 85%)

Allocation result			
Confidence Level	**95%**	**90%**	**85%**
Public (IG) Debt	38%	6%	0%
Public Equity	35%	65%	47%
LP Mezz. Debt	8%	1%	1%
LP Buyout	16%	26%	48%
LP Real Estate	3%	2%	4%
Expected Horizon Value	**196,930,799**	**292,784,752**	**342,704,324**
Horizon Value Volatility	136,179,356	232,198,811	270,700,539



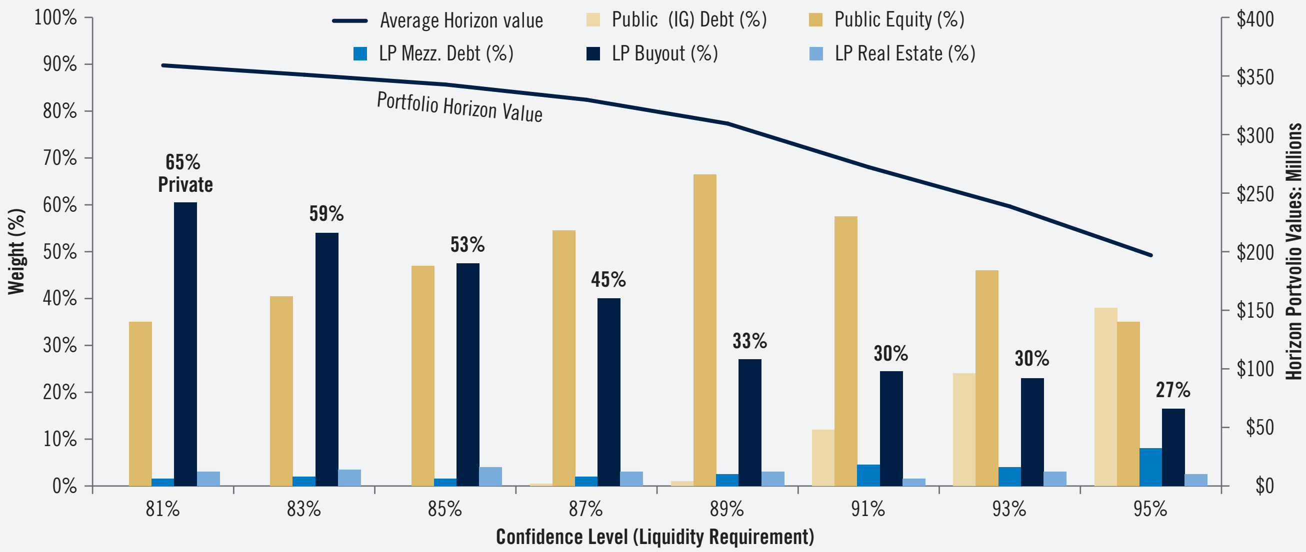
Note: Horizon value volatility is the standard deviation of horizon values across all the successful runs for the optimal asset allocation result.
Source: PGIM IAS. Provided for illustrative purposes only.

Measuring the Cost of Liquidity

Figure 5 depicts in more detail how the optimal asset allocation and the corresponding horizon portfolio value change with increasing confidence levels. Starting from a relatively low confidence level (*i.e.*, 81%), as the liquidity requirement increases, the allocation to private assets (as a percentage of the total portfolio) declines and the allocation to the risky public asset (S&P 500) increases. For example, moving from an 81% confidence level to an 89% confidence level the allocation to private assets falls from 65% of the total portfolio to 33%, while the allocation to public equity increases from 35% to 67%. Despite the large shift in asset allocation, the expected portfolio horizon value falls by $50m, from $359m to $309m. However, starting at an 89% confidence level, further increases in the liquidity requirement have a relatively muted effect on the public-private asset allocation, and more effect on the allocation within both the public and private portfolios. As the liquidity requirement increases, the allocation to the low-risk public asset (US IG debt) increases at the expense of the high-risk public asset (S&P 500). There is also some de-risking within the private portfolio (*i.e.*, less LP buyout). The performance impact of a higher liquidity requirement is a sharp reduction of portfolio horizon values. As the liquidity requirement increases the horizon value decreases, with steeper drops at higher liquidity requirement levels. This loss in horizon value as the liquidity requirement increases captures the “cost of liquidity”. Increasing the liquidity requirement from an 89% confidence level to a 95% confidence level reduces the portfolio's expected horizon value from $309m to $197m, a decline of $112m.

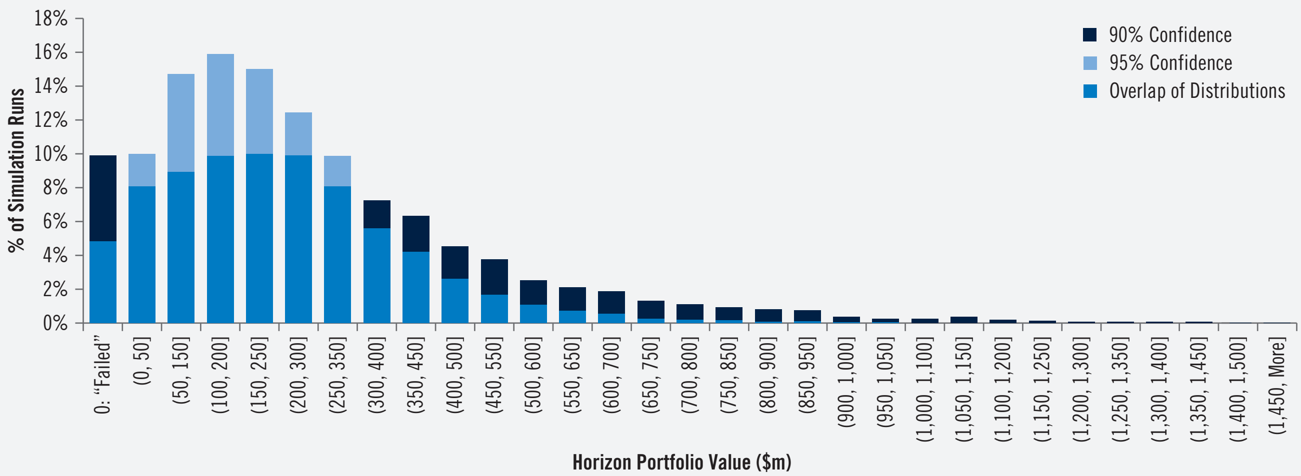
As mentioned, for each potential asset allocation solution, we simulate many scenarios (5,000). Each individual simulation run results in a different horizon value and we can determine the percentage of simulation runs that meet the liquidity requirement (*i.e.*, the confidence level). Hence, for a selected confidence level we can examine the distribution of horizon values stemming from the numerous corresponding simulation runs for a potential asset allocation solution. Figure 6 shows the distribution of possible

Figure 5: Portfolio Sensitivity to Changes in Confidence Levels (Liquidity Requirements)



Source: PGIM IAS. Provided for illustrative purposes only.

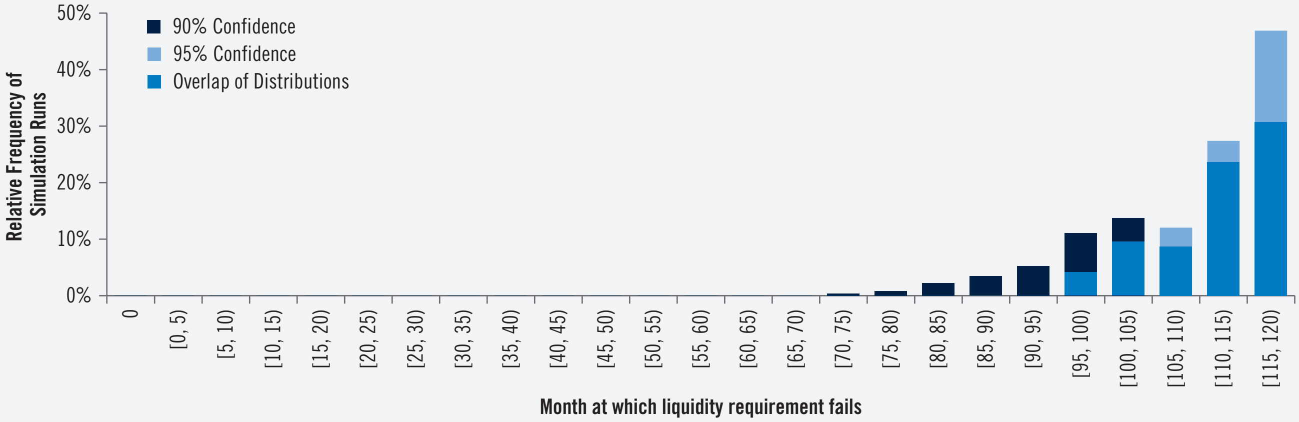
Figure 6: Horizon Portfolio Value Distribution
(Confidence Levels of 95% and 90%)



Note: The asset allocation used for this example is 6% public debt, 65% public equity, 1% LP mezzanine debt, 2% LP real estate and 26% LP buyout for a 90% confidence level; and 38% public debt, 35% public equity, 8% LP mezzanine debt, 3% LP real estate and 16% LP buyout for a 95% confidence level.

Source: PGIM IAS. Provided for illustrative purposes only.

Figure 7: Stopping Time Distribution
(Confidence Levels of 95% and 90%)



Note: This example shows, at 90% and 95% confidence levels, the distribution of the relative frequency of the first month when the portfolio is unable to pay the cash obligation. For each month, "relative frequency" is the number of failed simulation runs divided by the number of total failed simulation runs. "Stopping time" is only applicable to failed simulation runs. The asset allocation used for this example is 6% public debt, 65% public equity, 2% LP real estate, 1% LP mezzanine debt and 26% LP buyout for a 90% confidence level; and 38% public debt, 35% public equity, 3% LP real estate, 8% LP mezzanine debt and 16% LP buyout for a 95% confidence level.

Source: PGIM IAS. Provided for illustrative purposes only.

horizon values for two optimal asset allocations: One for a 90% confidence level and the other for 95%.[8] The distribution of horizon values for the 95% confidence level portfolio is shifted to the left of the other. This is the impact of the higher liquidity requirement. As liquidity requirements increase, the allocation to less risky and less rewarding public assets increases, reducing the expected horizon value and its volatility.

For a given asset allocation there is always a chance that the allocation may fail to satisfy all future cash obligations. The framework considers a simulation run a failure irrespective of when the portfolio fails (*e.g.*, in the first year, or in the last year). A CIO would likely want to know when, over the horizon, an asset allocation is unable to meet a liability. To answer this question we examine all failed simulation runs, for a given asset allocation, and keep a record of the first occurrence (*i.e.*, month) of a failure (*i.e.*, the "time of failure" or "stopping time") for each run.

Figure 7 shows the stopping time distributions for the two previously studied 90% and 95% confidence level optimal portfolios. For each month, it depicts the failed simulation runs in that month as a percentage of all failed simulation runs. It shows that a more liquid portfolio (*i.e.,* a higher confidence level) will have later stopping times. This is captured by the rightward shift in the stopping time distribution for the 95% confidence level portfolio relative to that for the 90% portfolio. For the 95% confidence level portfolio, the earliest stopping time is between month 95 and month 100.

"What if" Analysis

The framework allows the investor to perform "what if" analysis, *i.e.*, what is the impact on optimal asset allocation if one or more assumptions in the baseline scenario change, such as views on LP investment performance, or private asset transaction costs?

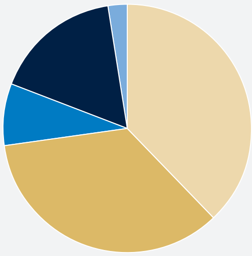
Suppose the investor has a negative view on future LP buyout investment (vintage-level) performance – relative to historical performance – but a positive view on LP mezzanine debt and LP real estate investments.[9] Figure 8 shows the effects of these views relative to the baseline asset allocation. As might be expected, the allocation to LP real estate rises from 3% to 7% and the allocation to LP mezzanine debt increases from 8% to 14%. Perhaps unexpectedly, the allocation to LP buyout also rises from 16% to 18%. While unexpected, the allocation is in fact in line with the expressed views. The higher allocation to LP real estate and mezzanine debt, and lower allocation to public equity, help lower overall portfolio volatility – providing some scope for an increased allocation to LP buyout. In addition, the asset allocation solution addresses not only the absolute level of allocations

8 Note that the percentage of time the horizon value is zero equals 1 minus the confidence level (%). This follows from our definition of the liquidity requirement. If we meet the liabilities at 90% confidence level, then in 10% of the 5,000 simulation runs the portfolio value fails to meet all of its cash obligations, leading to a zero horizon value.

9 See Appendix A3 for probability specification in each of four performance quartiles for each LP investment types.

Figure 8: Optimal Asset Allocation
Incorporating Investor Views on Future LP Performance

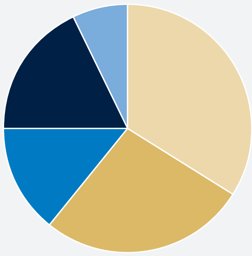
95% Confidence Level
Avg. Horizon Value = $197m
Volatility = $136m
(baseline)



38% Public (IG) Debt
35% Public Equity
8% LP Mezz. Debt
16% LP Buyout
3% LP Real Estate

	Investor Views on Future LP Performance		
LP Type	Buyout	Mezz. Debt	Real Estate
Investor Views	Negative View	Positive View	Positive View
Quartile 1 (Q1 - highest)	10%	40%	20%
Q2	20%	30%	40%
Q3	50%	20%	30%
Quartile 4 (Q4 - lowest)	20%	10%	10%

95% Confidence Level
Avg. Horizon Value = $204m
Volatility = $134m
(with views)

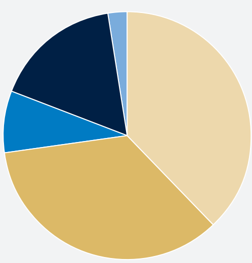


34% Public (IG) Debt
27% Public Equity
14% LP Mezz. Debt
18% LP Buyout
7% LP Real Estate

Source: PGIM IAS. Provided for illustrative purposes only.

Figure 9: Optimal Asset Allocation
Incorporating Investor Views on Future LP Performance & Fund-Selection Skill

95% Confidence Level
Avg. Horizon Value = $197m
Volatility = $136m
(baseline)

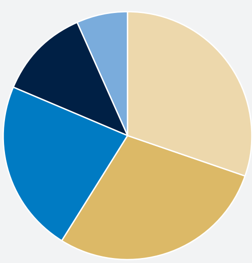


38% Public (IG) Debt
35% Public Equity
8% LP Mezz. Debt
16% LP Buyout
3% LP Real Estate

	Investor Views on Future LP Performance		
LP Type	Buyout	Mezz. Debt	Real Estate
Investor Views	Negative View	Positive View	Positive View
Quartile 1 (Q1 - highest)	10%	40%	20%
Q2	20%	30%	40%
Q3	50%	20%	30%
Quartile 4 (Q4 - lowest)	20%	10%	10%

	Investor Fund-Selection Skill		
LP Type	Buyout	Mezz. Debt	Real Estate
Fund-Selection Skill	Below-Average	Above-Average	Average
Quartile 1 (Q1 - highest)	10%	30%	25%
Q2	20%	40%	25%
Q3	50%	20%	25%
Quartile 4 (Q4 - lowest)	20%	10%	25%

95% Confidence Level
Avg. Horizon Value = $206m
Volatility = $141m
(with views and skill)



30% Public (IG) Debt
29% Public Equity
23% LP Mezz. Debt
12% LP Buyout
7% LP Real Estate

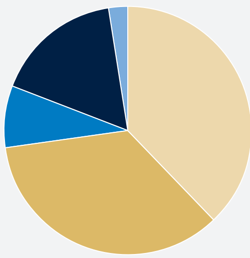
Weights may not add up to 100% due to rounding.

Source: PGIM IAS. Provided for illustrative purposes only.

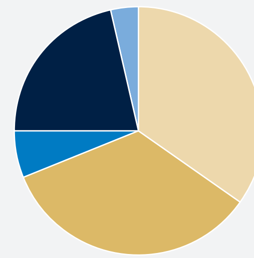
but also the inter- and intra-public and private allocations. In this example, the total portfolio allocation to private assets increases from 27% to 39%. As a component of the private asset portfolio, the LP buyout allocation falls, reflecting the relative views.

Figure 9 illustrates the effect on asset allocation of both the investor's fund-level selection skill and views on vintage-level LP investments. For example, an investor may have a view that LP mezzanine debt investments will outperform versus their historical

Figure 10: Optimal Asset Allocation
Lower Private Asset Transaction Costs



	LP Investment Transaction Costs (%)		
LP Type	Buyout	Mezz. Debt	Real Estate
	Baseline		
"good" economy	5%	5%	5%
"bad" economy	30%	15%	9%
	Lower Transaction Costs		
"good" economy	3%	3%	3%
"bad" economy	20%	10%	6%



Source: PGIM IAS. Provided for illustrative purposes only.

performance relative to public markets, and that they can select above-average LP mezzanine debt funds. In line with expectations, Figure 9 shows that these combined positive views and skill on LP mezzanine debt lead to an increase in allocation from 8% to 23%. In contrast, the combined negative outlook on LP buyout investments and below-average LP buyout fund-selection skill lead to a reduced allocation from 16% to 12%.

As another example of "what if" analysis, Figure 10 shows the impact of lower private asset transaction costs on the optimal asset allocation. At a 95% confidence level, lower private asset transaction costs (with public asset transaction costs unchanged) lead to an increased allocation to private assets. Specifically, the allocation to LP buyout increases from 16% to 22%, with an unchanged allocation to LP real estate. This is expected as lower private asset transaction costs mean a lower value penalty when these assets need to be sold, hence a higher probability of meeting the liquidity requirement. This increase in portfolio liquidity allows a higher allocation to private assets which have higher expected returns than public assets.

This Case Study illustrates how a hypothetical investor could use this framework to evaluate their "cost of liquidity", in terms of their expected future horizon portfolio value, when considering adding an allocation to private assets. In the next section, we shift attention to the framework's detail and describe the key components and methodologies underlying the framework.

Asset Allocation Framework – Detail

As mentioned in the Case Study, the three key components of the asset allocation framework are: the investment opportunity set comprising both public and private assets; the investor's investment objective; and a periodic (monthly or quarterly) liability schedule. The framework is flexible to accommodate different investment objectives and assets types. To illustrate how the framework works we adhere to the objective of maximizing expected horizon portfolio value with five asset types (S&P 500, Bloomberg Barclays 5-10y US IG Corporate Index, LP buyout, LP mezzanine debt, and LP real estate), as introduced in the Case Study.

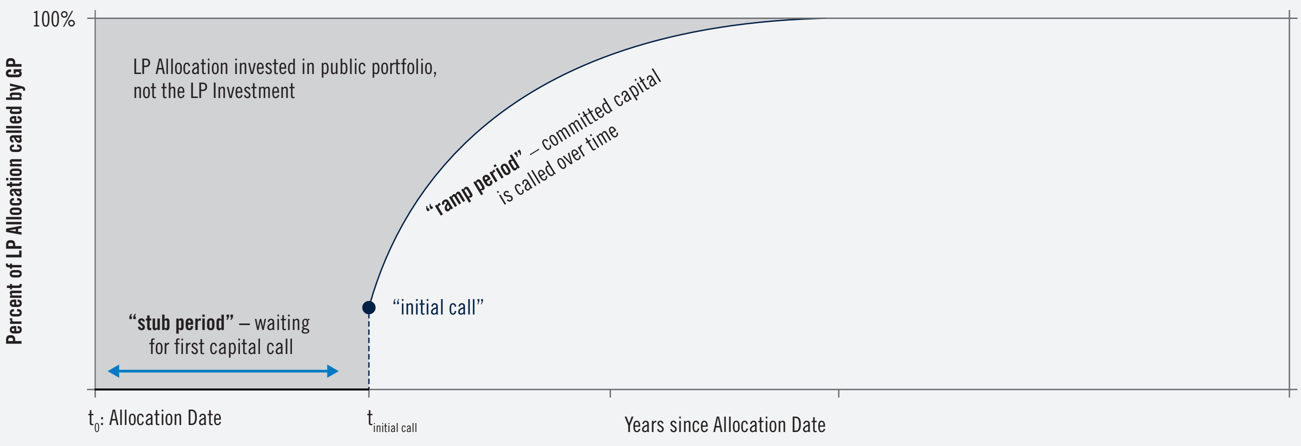
This section provides step-by-step details on how the framework connects the three main components to determine the optimal asset allocation. Specifically, the steps are:

1. Simulate future public and private asset values;
2. Assess the portfolio's liquidity adequacy over the entire investment horizon; and
3. Determine the optimal asset allocation.

Simulating Public and Private Asset Values

The framework relies on simulation to estimate asset values over time. For each potential asset allocation solution, we conduct 5,000 simulation runs. Each simulation run generates monthly return series for two public assets up to the horizon, and 120 monthly interim values for three private assets. For public assets, monthly returns are drawn from a joint t-distribution for

Figure 11: Pattern for the Investment Composition of an LP Allocation over Time



Source: PGIM IAS. Provided for illustrative purposes only.

the entire investment horizon. Note that the characteristics of the t-distribution are determined by an investor's public market assumptions (Figure 3). To estimate private asset values, we model the co-movements linking public and private assets' holding period values.

Private asset valuations are challenging due to a lack of high frequency price and return data, unlike the case for public assets. We propose a methodology for estimating private investments' periodic values, rather than trying to estimate time-weighted returns. We also explicitly account for the gradual and uncertain timing of committed capital calls in private markets.

For private assets, we distinguish between ***LP allocation*** and ***LP investment*** values by including in the former the horizon value of any undrawn capital – assuming any undrawn capital is invested in a "default public investment" until called by the GP. The horizon value of an LP allocation or an LP investment is affected by the timing and magnitude of the capital calls. The initiation and subsequent timing of LP capital calls is at the GP's discretion. Figure 11 provides a stylized illustration of the timing of capital calls for an LP investment. Following an allocation (*i.e.*, a commitment) at time t_0, there is a period, which we label as the "stub period", until the GP makes the first capital call (time $t_{\text{initial call}}$). Once the GP initiates calls, it may take years – the "ramp period" – until all the committed capital is called.

Figure 11 shows that an investor's LP allocation is a combination of the LP investment itself and a "default public investment". During the stub period the LP allocation is 100% invested in the default investment, then the uncalled committed capital decreases as the GP makes capital calls. The investor faces the tradeoff between investing uncalled capital in a low-risk public asset foregoing potentially higher horizon portfolio values for the LP allocation, and investing in a high-risk public asset but exposing the portfolio to liquidity risk (*i.e.*, the risk the investor not being able to meet capital calls due to poor performance of the default investment). For simplicity, we assume the default investment is what comprises the public portfolio in an asset allocation solution. For example, if the potential asset allocation starts with 20% public equity, 30% public (IG) debt, and 50% LP buyout, then the default investment is a public portfolio composed of 40% public equity and 60% public (IG) debt.

LP Investment Value

To estimate LP allocation values, we first need to estimate the LP investment value. To do so, we use the public-market-equivalent (PME) ratio to connect the performance of private assets and public assets.[10] PME is a market-adjusted cash multiple. Mathematically, it is the ratio of the horizon value of cash received (distributions) to the horizon value of cash paid (*i.e.*, contributions/capital calls) assuming they are each invested in a benchmark public portfolio.[11] The framework uses the S&P 500 Index as the benchmark. PME measures the value multiple effect of investing in the LP investment versus the selected benchmark. A PME greater than 1 represents a horizon value of the LP investment greater than the horizon value of investing the capital calls in the S&P 500 Index.

The LP investment value is defined as the horizon value of distributions (assuming distributions are reinvested in S&P 500), which is the numerator of the PME. Therefore, the LP investment horizon value can be calculated by multiplying an estimated PME

10 For more information on PME, please refer to S. Kaplan and A. Schoar, "Private Equity Performance: Returns, Persistence, and Capital Flows", *Journal of Finance*, Vol. LX, No.4, August 2005.
11 Details about the PME calculation can be found in Appendix A4.

(from a simulation run) by the horizon value of an S&P 500 investment strategy assuming capital is invested in the strategy based on the same LP investment capital call schedule. Consequently, our two estimation steps are:

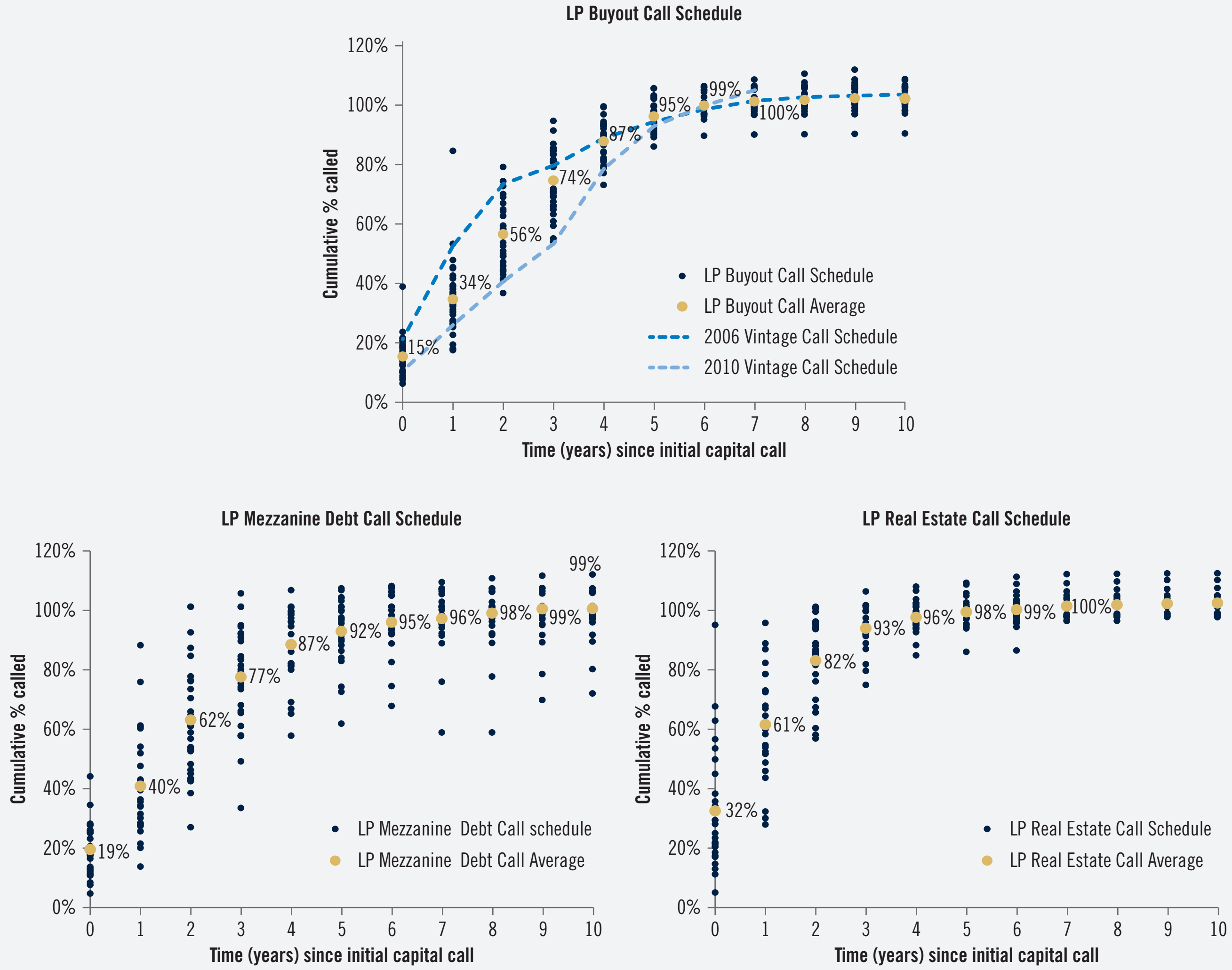
1. Estimate the horizon value of the S&P 500 investment strategy assuming capital is invested based on the same LP investment call schedule. To do this, we establish a model of the capital call schedule to measure the expected yearly call amounts; and
2. Estimate the PME value from simulated public assets holding period returns, based on the empirical relationship between public asset returns and PME values.

Appendix A7 contains a detailed example of the LP investment value calculation.

Estimating the capital call schedule

As mentioned, the potentially delayed and gradual drawdown of committed capital can affect the performance measurements of the funds (both LP investment values and LP allocation values). To allow for more realistic estimates of the LP investment and LP allocation values we need to estimate the call schedule. The estimation accounts for the impact of general market conditions where we observe a difference in the pattern of capital calls conditioned on market conditions.

Figure 12: "Ramp Period" – Capital Call Schedule after Stub Period
(for LP buyout, LP mezzanine debt and LP real estate funds)



Source: Burgiss, PGIM IAS. Provided for illustrative purposes only.

Figure 12 illustrates the pattern and variability of vintage-level call schedules for the three LP types.[12,13] The data do not provide information on the length of the stub period. Hence, the framework requires investors to provide an estimate. Once capital calls begin, the data (Burgiss) show that calls extend over many years, allowing uncalled capital invested in the default portfolio to grow. This not only provides, for a time, some liquidity to the overall portfolio, but also may increase the LP allocation value compared to the stand-alone LP investment value.

In general, Figure 12 shows that the percentage of capital called up to a given year is a positive, generally increasing curve which approaches a limit of around 100%. To model this pattern of the call schedule we use a logistic function:

$$f(t) = \frac{1}{\left(1 + \left(\frac{1}{a} - 1\right) e^{-bt}\right)} \tag{1}$$

where, t represents the time (in years) since the initial capital call.

In addition to this general call schedule specification, the estimated call schedule for each LP investment type needs to incorporate the observed dependence of calls on public market conditions. For example, if public market conditions are poor, the data indicate that the initial call is typically lower than average. To illustrate, the LP buyout call schedule of Figure 12 shows that the LP buyout 2006 vintage had a higher initial capital call than the 2010 vintage that followed the financial crisis. It is also clear that the 2010 vintage catches up with 2006 vintage in terms of being fully called within the same number of years. To capture the variation in the call schedule, we let the parameters, a and b, which govern the initial level and speed, respectively, of capital deployment depend on lagged S&P 500 Index total returns.[14] Specifically, we let

$$\begin{aligned} a &= a_0 + a_1 \times TR_{SPX}(-2,-1) + a_2 \times TR_{SPX}(-3,-2) \\ b &= b_0 + b_1 \times TR_{SPX}(-2,-1) + b_2 \times TR_{SPX}(-3,-2) \\ TR_{SPX}(-2,-1) &= \frac{SPX_{t=-1} - SPX_{t=-2}}{SPX_{t=-2}} \\ TR_{SPX}(-3,-2) &= \frac{SPX_{t=-2} - SPX_{t=-3}}{SPX_{t=-3}} \end{aligned} \tag{2}$$

where, for example, $TR_{SPX}(-3,-2)$ represents the cumulative total return on the S&P 500 Index for the two years prior and the three years prior to the first capital call.

Note that f(0) = a, and given positive estimates for parameters a_0, a_1, and a_2, positive market returns before the initial capital call signify higher initial capital calls. Negative values for parameters b_1 and b_2 indicate the catch-up tendency of low market return vintages. Appendix A5 shows that for LP buyout the estimates for a and b are in line with the observations in Figure 12, positive for a and negative for b.[15]

Estimating vintage-level PMEs

We first estimate PMEs at the vintage level, then at the fund level. Both vintage-level and fund-level PMEs are drawn from estimated distributions building on observed patterns in the historical data. We derive the expected vintage-level PMEs, ($\hat{PME_v}$), using an estimated historical relationship between PMEs and public market returns. Historically observed horizon pooled vintage-level PMEs are regressed on public market holding period total returns. Both the PMEs and public asset values are measured over the same period.[16] The public assets are the public equity S&P 500 (SPX) and public US High Yield debt (Bloomberg Barclays US HY Index). The regressions are estimated separately for each LP investment type and interim holding periods over the 10y investment horizon. Results indicate statistically that only LP buyout PMEs depend on both SPX and US HY. LP mezzanine debt and LP real estate depend on SPX only.[17]

12 Data are from Burgiss. The Burgiss data cover complete transactional data for a large number of limited partnership funds. All data are sourced exclusively from the limited partners. (https://privateiq.burgiss.com/Resources/pdf/Essential%20Facts.pdf). Other data sources do exist but their sources are not exclusively from the limited partner. Preqin obtains much of its data through involuntary Freedom of Information Act (FOIA) requests and Cambridge Associates obtains its data from LPs and GPs. See Harris, *et al.,* "Private Equity Performance: What Do We Know?", *Journal of Finance*, Vol 69, Issue 5, March 2014.

13 A fund is part of a given vintage (*i.e.*, calendar year) if the fund's first capital call is made in that year. For example, the 2006 vintage consists of all funds who had their first capital call in 2006. Hence, the data do not permit us to estimate the length of the "stub" period. The framework addresses the stub and ramp periods separately.

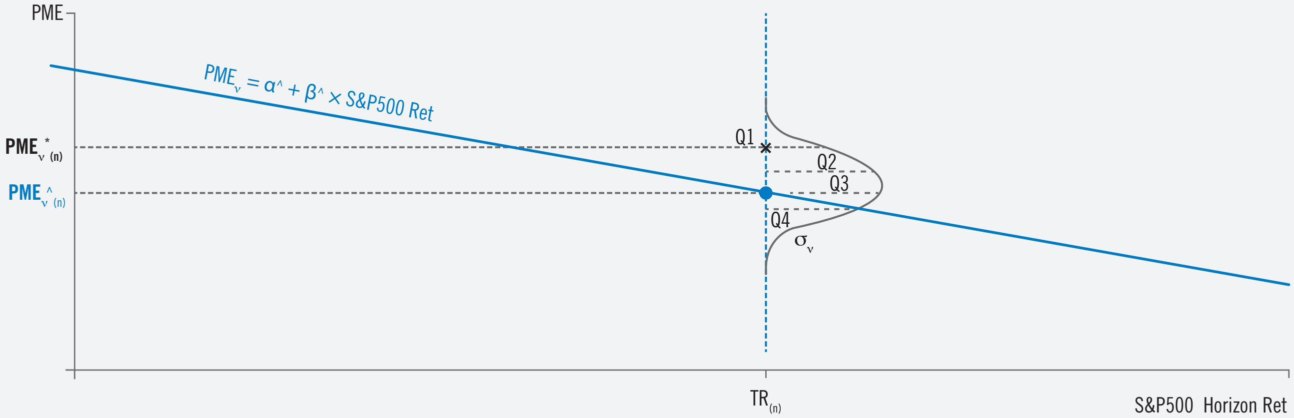
14 Appendix A5 shows parameter estimates fitted to the logistic function for each LP asset type.

15 We note that for LP mezzanine debt the estimated parameter values are negative for *a* and positive for *b*.

16 "Pooled" PME combines all the funds of a given vintage into a single fund before computing the PME. A pooled PME will give more weight to larger funds. "Average" PME is the equally-weighted arithmetic average PME across funds.

17 Appendix A6 gives detailed specification (for each LP investment type) of the pooled vintage-level 10y PME regression on public asset 10y annualized total returns.

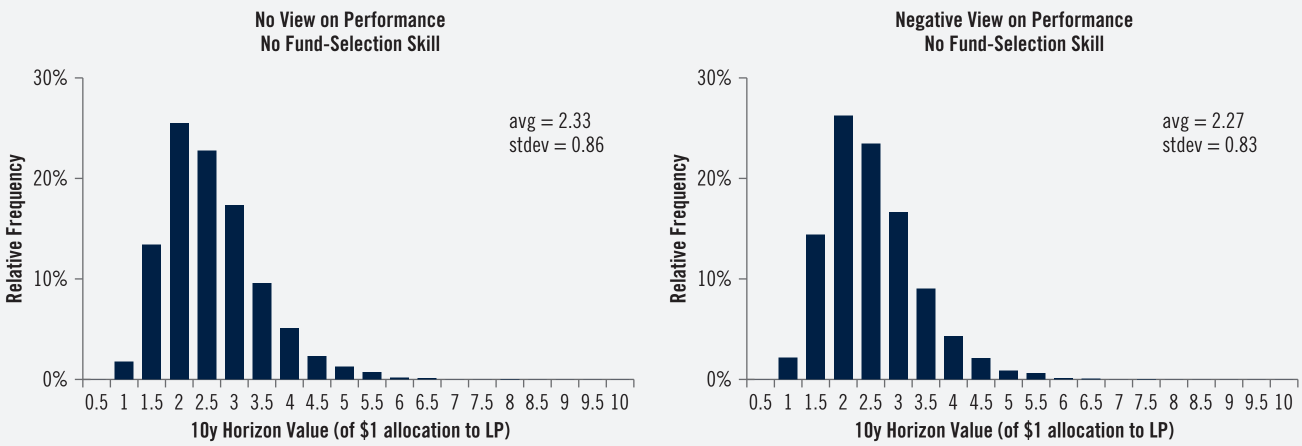
Figure 13: Estimating a Vintage-Level PME for a Simulation Run



Note: For simplicity, this figure assumes a single variable model of PMEs (*i.e.*, S&P 500 Index). In practice, we model PMEs of LP buyout investments using a multi-variable model of public asset returns. $TR_{(n)}$ refers to total return of the n^{th} simulation run.

Source: PGIM IAS. Provided for illustrative purposes only.

Figure 14: Impact of LP Investment Performance View on 10y Horizon Value Distribution



Note: The distributions presented in this figure represent 10y horizon values (of $1 allocation to LP buyout funds) with a 2y stub period. Assumed probabilities assigned to each of the four quartiles reflecting a negative view on LP buyout performance in this illustration (right panel) are: Q1 = 10%, Q2 = 20%, Q3 = 50%, and Q4 = 20%.

Source: PGIM IAS. Simulations provided for illustrative purposes only.

Using LP buyout at the 10y horizon as an example, we have:

$$10y\ PME_v^{B/O^{\wedge}} = 1.09 - 3.77 \times TR_{SPX}(0,10) + 5.50 \times TR_{HY}(0,10) \quad (3)$$

Equation (3) shows that vintage-level PMEs are positively related to the 10y US High Yield holding period total return. The positive relationship with the US HY total return index suggests a cost-of-funding channel for boosting LP buyout performance. Perhaps surprisingly, we observe a negative relationship between PME and 10y S&P 500 Index holding period total returns. This holds true for all three LP investment types (see Appendix A6). It indicates that the relative outperformance of LP investments is lower when the public markets are exhibiting higher returns.[18] In contrast, when public equity markets do less well, LP investments show a tendency to perform relatively better.

For each simulation run, we select a PME value for each horizon period and for each LP investment type. Figure 13 illustrates the process for LP buyout, which assumes, for simplicity, that vintage-level PMEs are a function only of the S&P 500 Index total

18 The PMEs are based on the LP investor's net-of-fees cash flows. The reduction in cash flows induced by fees might be contributing to the observed negative relationship between PMEs and 10y S&P 500 Index total returns.

return. For a given simulation run (call it “n”) we have an S&P 500 Index total return ($TR_{(n)}$). The downward sloping regression line depicts the possible expected PME values implied by a given S&P 500 Index horizon total return. The blue dot on the line is the expected vintage-level PME, $PME^{\wedge}_{v\,(n)}$ given the S&P 500 Index horizon return from the simulation run. Due to prediction error, the true expected PME value lies within a distribution centered around $PME^{\wedge}_{v\,(n)}$. For a simulation run, the vintage-level PME, ($PME^{*}_{v\,(n)}$), is drawn from a Normal prediction error distribution centered around $PME^{\wedge}_{v\,(n)}$: $N(10y\ PME^{\wedge}_{v\,(n)},\ 10y\ \sigma_{v})$.

The vintage-level PME can be drawn from different parts of the prediction error distribution in accordance with an investor’s view on LP investment performance relative to a public asset benchmark (*e.g.*, S&P 500). If an investor believes that future LP buyout investments are likely to experience below-average relative performance, they can choose to give more weight to the lower quartiles (*i.e.*, Q3 and Q4) of the distribution. In other words, the simulation will be more likely to draw a “negative” prediction error. Figure 14 shows the reduction in 10y LP buyout allocation values, from 2.33 to 2.27, resulting from a negative view. Note that a vintage-level LP allocation does not involve the investor’s skill in LP fund selection.

PME quality consistency over time

Estimating vintage-level PME_{v}^{*} values for all earlier horizons must be carried out to fully capture an LP investment’s value over time. However, this must be done in a consistent way. In other words, if in a simulation run an LP investment in year 10 is of high quality (*i.e.*, an above-average PME), then the LP investment would most likely be of high quality in year 9. **“Quality consistency”** is ensured throughout the life of a simulation run, from year 1 to year 10, by letting an LP investment’s quality to gradually evolve to a specific 10y LP investment quality. Hence, the procedure works backwards in time, starting at the 10y horizon where the final LP investment quality is determined, then using quality-quartile transition matrices to work backwards in time.

To ensure quality consistency, we estimate two quality-quartile transition matrices – one for the first five years and the other for the remaining five years. This follows the observation of a large shift in the transition probabilities across the two periods. Figure 15 shows that if an LP investment is in quality quartile Q1 in year 10, then the Year 10 to Year 6 transition matrix says that there is a 90% probability that in year 9 the LP allocation was in Q1 and only a 2% probability it was in Q3. The shift in the magnitude of the diagonal elements between the two matrices highlights that the probability of remaining in a given quality quartile is high (*i.e.*, 84% - 90%) during the last 5 years. In contrast, during the first 5 years the probability of remaining in a given quality quartile is much lower (*i.e.*, 40% - 48%), making it more likely that an LP investment may transit across quality quartiles before their fifth birthday, but less so afterwards.

Ensuring quality consistency complicates the simulation of LP investment performance. Suppose in year 10, a simulation produces a PME at point A (Figure 16). Point A is a Q1 LP investment. Figure 16 also shows the 9y S&P 500 return for this simulation run and the PME distribution. Since A was in Q1 in year 10, for year 9 the simulation run applies probabilities from the quality-quartile transition matrix (*i.e.*, 90% for Q1; 7% for Q2; 2% for Q3; and 1% for Q4) when sampling from the PME prediction error distribution – producing a year 9 PME value for the simulation run at point B.

Figure 15: “Backward” LP Investment Quality-Quartile Transition Matrices

Year 10 to Year 6

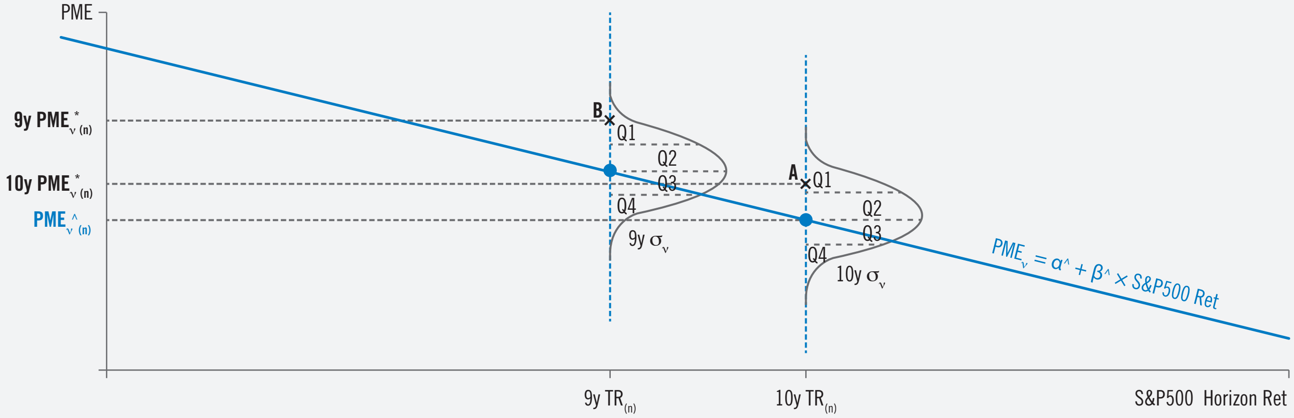
Year (Y-1) Quartiles	Year (Y) Quartiles Q1	Q2	Q3	Q4
Q1	90%	7%	2%	1%
Q2	7%	84%	7%	2%
Q3	2%	7%	84%	7%
Q4	1%	2%	7%	90%

Year 5 to Year 1

Year (Y-1) Quartiles	Year (Y) Quartiles Q1	Q2	Q3	Q4
Q1	48%	22%	16%	14%
Q2	22%	40%	22%	16%
Q3	16%	22%	40%	22%
Q4	14%	16%	22%	48%

Source: PGIM IAS. Provided for illustrative purposes only.

Figure 16: Incorporating "Quality Consistency" in Estimating PMEs



Note: For ease of presentation, this figure assumes a single variable model of PMEs (*i.e.*, S&P 500). The 9y PME regression is different from 10y regression. For ease of illustration, we use a single downward sloping line to represent both.

Source: PGIM IAS. Provided for illustrative purposes only.

Incorporating quality consistency in the simulation produces two benefits:

1. Produces a more accurate reflection of an LP investment's lifetime performance; and
2. Allows the investor to incorporate a view on the private market premium at the outset of the simulation.

Estimating fund-level PMEs

Fund-level PMEs, PME_f, are also drawn from a Normal prediction error distribution. This distribution is centered around the estimated vintage-level PME, PME_v*, but with a variability based on the historically observed fund-level PME variation (σ_f). The resulting distribution is $N(PME_v^*, \sigma_f)$.

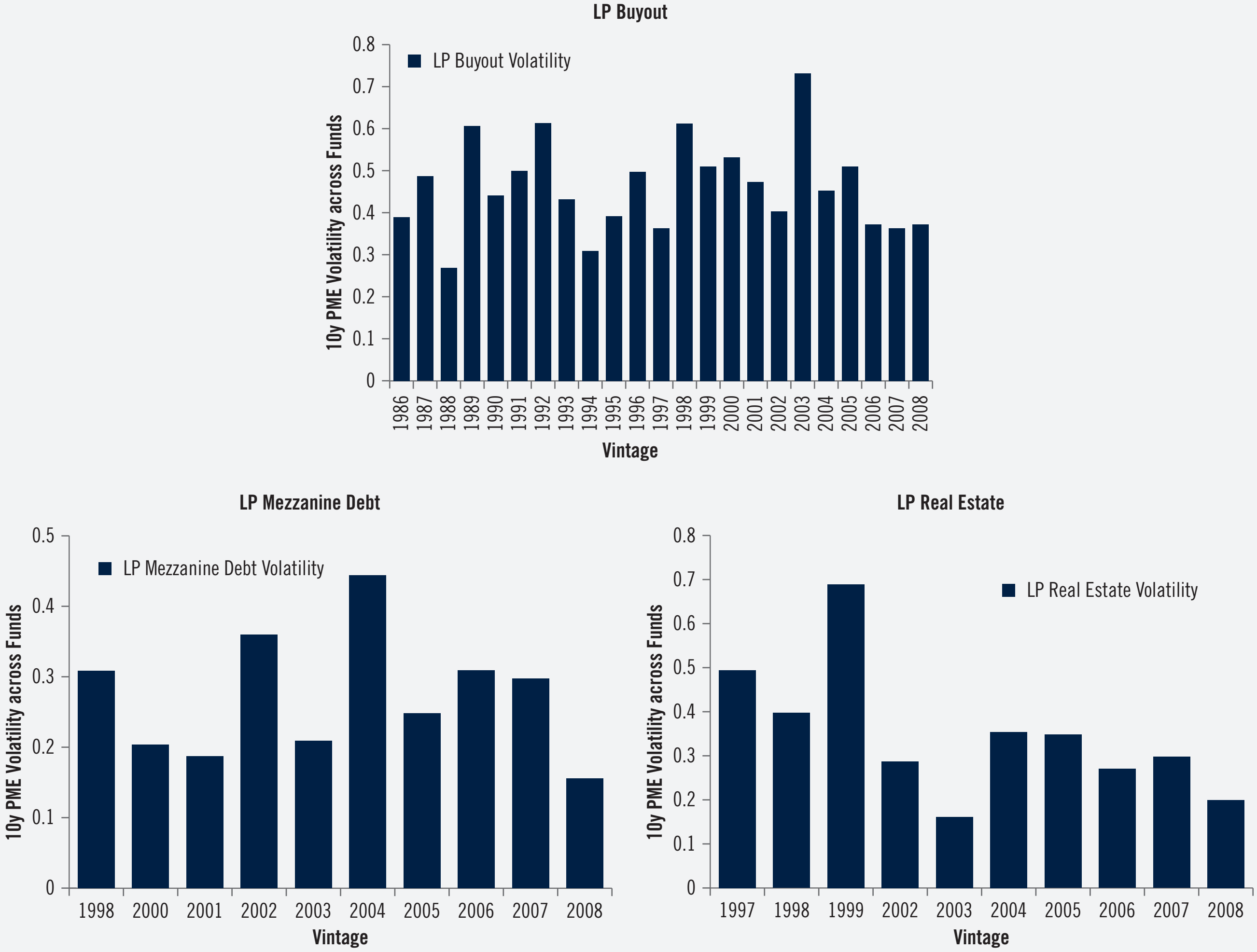
Figure 17 illustrates the variation observed in fund-level PMEs across vintages for the three LP investment types. For the 1986 to 2007 vintages the average cross-sectional 10y PME standard deviation for LP buyout was 0.46. Given an average of 10y vintage-level PME_v of only 1.19, fund-level PMEs can be relatively volatile. For the other two LP types we find a lower level of cross-sectional variability: for LP mezzanine debt (0.27 fund-level standard deviation averaged across the vintages, with a mean 10y vintage-level PME of 1.11); and for LP real estate (0.35 fund-level standard deviation averaged across the vintages, with a mean 10y vintage-level PME of 1.13).

Figure 18 shows the steps of estimating fund-level PMEs. Vintage-level data contains information about the underlying funds. We maintain the vintage-fund linkage by having a distribution of fund-level PME, PME_f, that has a mean of $PME^*_{v\ (n)}$, but a standard deviation of σ_f. The orange bell curve illustrates such a fund-level PME distribution. A simulation run produces an estimated $PME^*_{f\ (n)}$ by sampling (with replacement) from $N(PME^*_{v\ (n)}, \sigma_f)$. One possibility is marked by the orange "x".

As with vintage-level LP performance, investors can express their views on fund-level performance by specifying a quartile from the fund-level PME prediction error distribution. For example, an investor may express their below-average fund-selection skill by giving more weight to the lower quartiles of the distribution. Figure 19 shows the impact of such a choice. The 10y LP allocation value is reduced from 2.27 to 2.19 after the investor expresses below-average fund-selection skill.

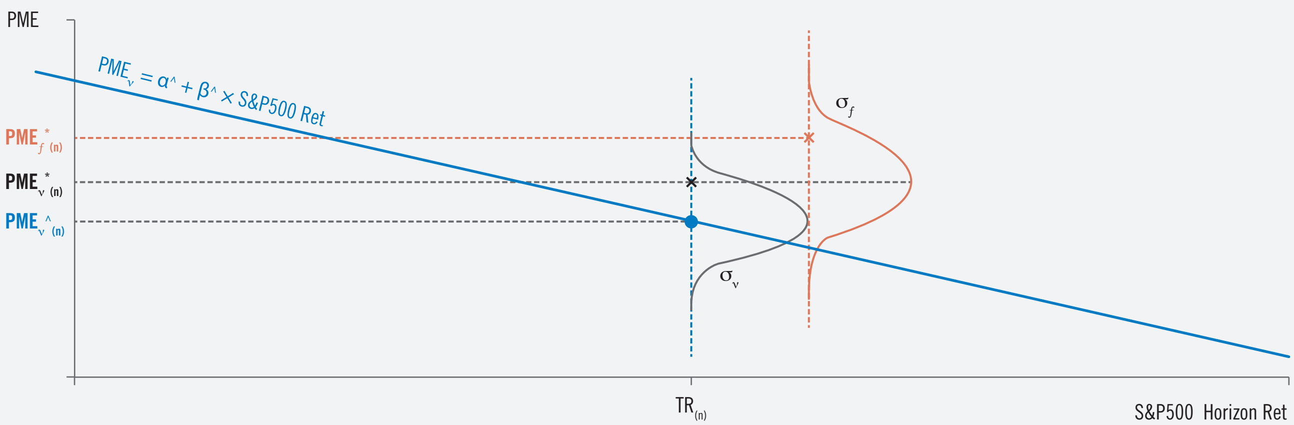
To produce quality-consistent PME_f values for other horizon periods, the framework follows the same procedure as described earlier for vintage-level PMEs using the same vintage-level quality-quartile transition probabilities.

Figure 17: Cross-Sectional Volatility of Fund-Level 10y PME_f by Vintage Year



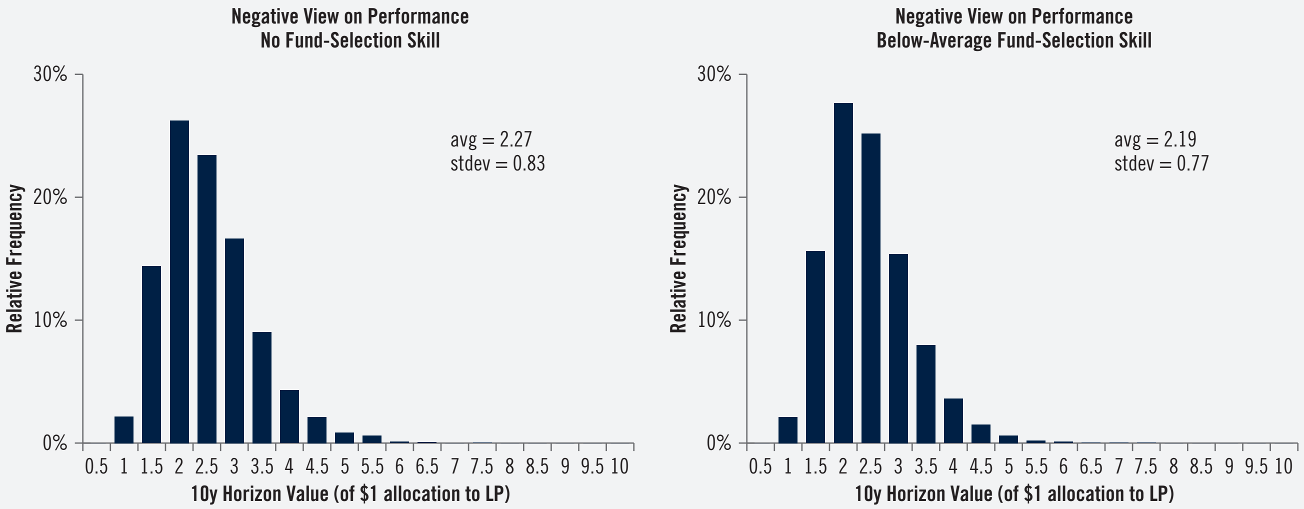
Source: Burgiss, PGIM IAS. Provided for illustrative purposes only.

Figure 18: Estimating a Fund-Level PME for a Simulation Run



Source: PGIM IAS. Provided for illustrative purposes only.

Figure 19: Impact of Investor's LP Fund-Selection Skill on 10y Horizon Value Distribution



Note: The distributions represent 10y horizon values (of $1 allocation to LP buyout investments) with 2y stub period. Assumed probabilities assigned to each of the four quartiles for below-average fund-selection skill are: Q1 = 10%, Q2 = 20%, Q3 = 50%, and Q4 = 20%.

Source: PGIM IAS. Simulations provided for illustrative purposes only.

LP Allocation Value

Having estimated an ***LP investment*** value we can now estimate the corresponding ***LP allocation*** value which incorporates the effect of undrawn amounts during the stub and call periods. The LP allocation value, rather than the LP investment value, directly reflects what the investor will likely experience at the time of making their asset allocation decision. The framework assumes that the undrawn capital is invested in the investor's public market portfolio (S&P 500 and 5-10y US IG, weights depending on the initial asset allocation). The LP allocation value is determined as follows:

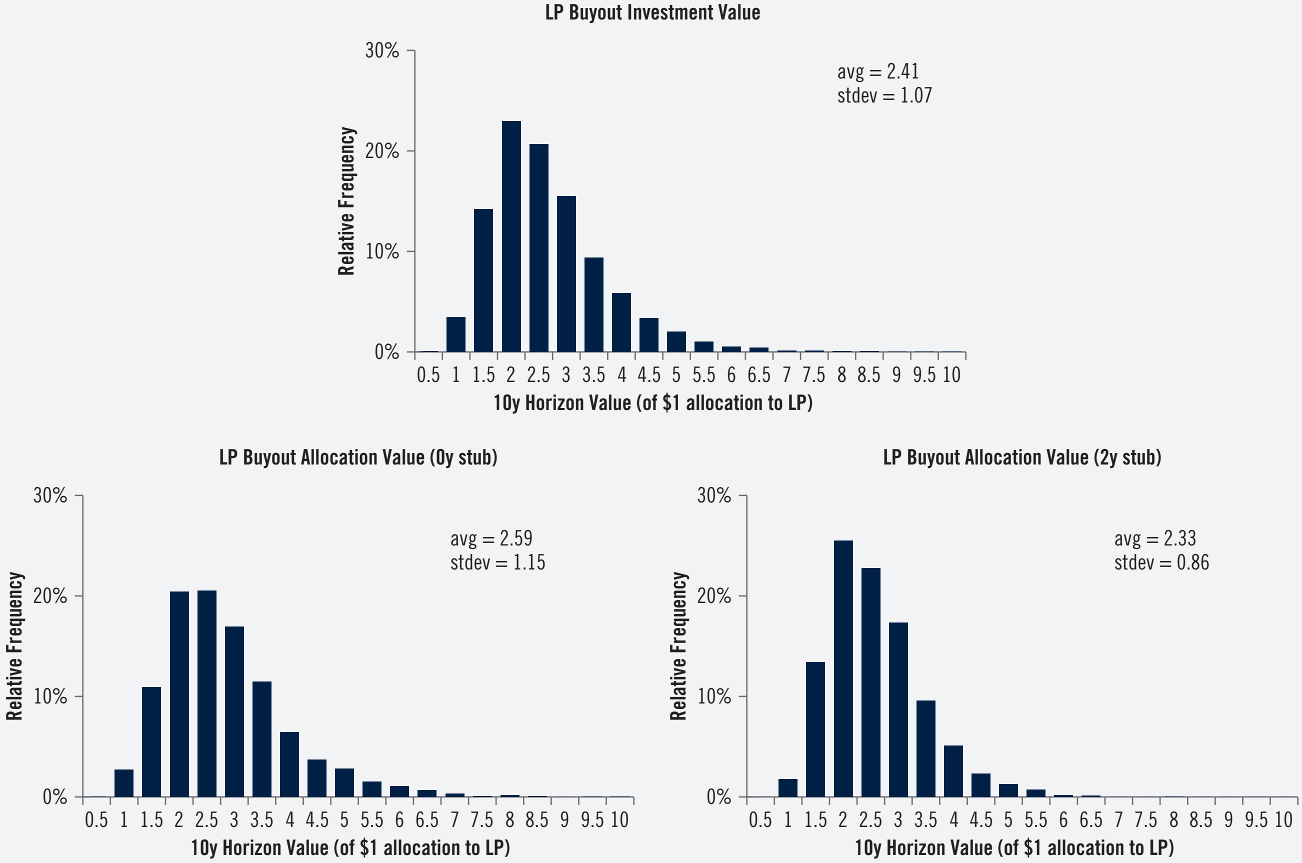
est. LP allocation value = LP investment value
+ future value of the public portfolio from the "stub" and "ramp" periods

$= PME_f^* \times$ future value of capital calls invested in an S&P 500 strategy
+ future value of the public portfolio from the "stub" and "ramp" periods (4)

Appendix A8 provides a detailed example of the LP allocation value calculation.

For LP buyout investments, Figure 20 depicts the difference between LP investment value and LP allocation value. Assuming no stub period, the increase from a 2.41 10y horizon value of LP buyout investment to a 2.59 10y horizon value of LP buyout allocation reflects, in this case, the added value from investing uncalled capital in the public portfolio. However, if there is a 2y delay of the first LP buyout investment capital call (*i.e.*, stub = 2y), the 10y LP allocation horizon value falls to 2.33 from 2.59, driven by loss from shortening the effective investment period of the LP buyout investment from 10y to 8y, which is partially offset by performance gain from investing in the default public portfolio.

Figure 20: LP Buyout Investment Value *vs.* LP Buyout Allocation Value



Note: This example assumes the uncalled capital is invested in a public market portfolio that is 50% S&P 500 and 50% 5-10y US IG bonds.

Source: PGIM IAS. Simulations provided for illustrative purposes only.

Assessing the Portfolio's Liquidity Adequacy

The estimates of public and private asset (LP allocation) values determine the horizon and interim portfolio values which allow us to assess the portfolio's capability to meet an investor's liquidity requirement.

For each portfolio there are 5,000 simulations runs, in effect producing 5,000 120-month series of interim monthly portfolio values. In a given month, if the portfolio cannot meet the cash obligation after transaction costs, then that simulation run stops and is labeled a **"failure"**.[19] If across the 5,000 simulation runs the percentage of "failures" is greater than or equal to one minus the investor's confidence level, then that potential asset allocation mix is considered **"infeasible"**. For example, if the investor specifies a 90% confidence level, then for at least 90% of the 5,000 simulation runs (*i.e.*, ≥ 4,500) a "feasible" asset allocation must always meet the cash flow obligations over the entire horizon.

For each feasible asset allocation, we calculate the expected horizon allocation value using the average horizon value across all successful simulation runs.

So far, we have assumed a deterministic cashflow obligation schedule. However, cash flow schedules can be subject to uncertainty due to factors such as changing inflation rates, variations in retirement age, and mortality risk.[20] If this cash flow uncertainty can be modeled by the framework's state variables, then this uncertainty can be incorporated in the optimal asset allocation solution.

19 A selling priority rule is proposed based on the illiquidity of different assets. Briefly, low transaction cost assets are sold first. In addition, private assets cannot be sold in the first several years of fund initiation in a good economy environment. See Appendix A2 for details.

20 IAS's upcoming paper on "Asset Allocation for 'End-State' Portfolios" illustrates asset allocation for "end-state" portfolios incorporating uncertainty in cash liabilities due to mortality risk.

Determining the Optimal Asset Allocation

The asset allocation framework examines a full breadth of possible asset allocations with 5% weight increments in each asset. The previously described process to determine the feasibility of an asset allocation solution is conducted for each of these possible asset allocations.

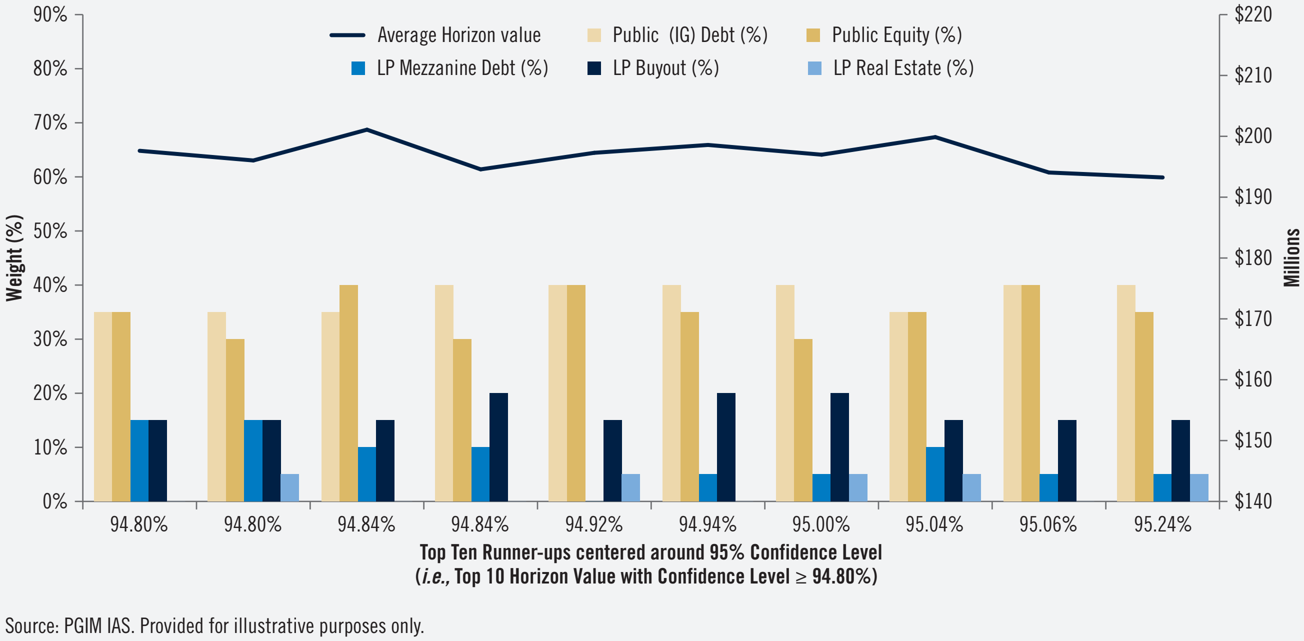
The optimal allocation solution corresponds to the portfolio that has the highest expected horizon value among all feasible solutions. However, due to the inherent randomness of simulations, one single simulation might identify an asset allocation as a feasible solution whereas another simulation might not depending on whether the number of failures is above or below the confidence level cut-off value. Consider two simulations of 5,000 runs and a confidence level of 90%. A given asset allocation can have 499 failures in one simulation and 501 in the other. In the second case the allocation is infeasible.

To alleviate the issue of optimal candidate portfolios bouncing in-and-out of the feasible solution set, we allow for some wiggle room and expand the set of feasible solutions to include those just below the specified confidence level. We then identify the optimal allocation as the average of the ten allocations with the highest horizon value. Choosing ten allocations allows for an even distribution of allocations above and below the specified liquidity level without foregoing convergence or missing out on reasonable allocations close to being feasible.

As an example, Figure 21 shows the ten potential allocation solutions that have the highest horizon values among all feasible solutions with a realized percentage of successful simulation runs starting at, in this case, 0.20% below the specified 95% confidence level. The average allocation across these ten portfolios is the optimal asset allocation solution presented in Figure 4 in the Case Study.

Notice that for each potential allocation (10,626 in total – 5% weight increments for each of the five assets) we need to determine the liquidity adequacy for every simulation run (5,000 in total). This implies a total of 53m simulation runs to identify the optimal allocation result — a large computational burden. If we were to permit a 2.5% weight increment in each asset, the number of potential allocations that must be examined increases to 135,751, implying approximately 680m simulation runs. Therefore, we adopted a hyper-scale cloud computing platform that significantly improves our modeling efficiency.

Figure 21: Ten Candidate Portfolios with the Highest Horizon Values at 95% Specified Confidence Level



Source: PGIM IAS. Provided for illustrative purposes only.

Summary

Private assets may provide a diversification benefit and a potential "private market premium" over public assets. However, investors have a concern whether an allocation to less-liquid private assets may render their portfolio unable to satisfy future cash liabilities as needed.

When making the asset allocation decision, an investor must balance the desire to attain higher returns against the need to meet obligations. In the context of an investment universe containing assets with widely varying liquidation characteristics, we argue that portfolio "liquidity" for an investor means the degree of confidence the investor has in meeting cash flow obligations across different economic scenarios. We propose an asset allocation framework that captures the investor's thought framework – "Maximize horizon portfolio value provided I am sufficiently confident of meeting my cash flow obligations".

We propose a simulation-based asset allocation framework that incorporates the performance of common private assets (limited partnerships, or LPs). We also recognize some unique characteristics of LP investments: possible delays for the first capital call; intermittent capital calls thereafter; higher and lumpier transaction costs relative to public assets. In addition, we allow investors to provide their views on LP performance and fund-selection skills.

The framework helps investors determine their optimal allocation across both private and public assets, and the sensitivity of the optimal allocation to changes in assumptions about the risk and return characteristics of both private and public assets (*e.g.*, performance views and transaction costs). The framework highlights that the asset allocation between private and public assets, as well as the asset allocation within the private and public portfolios themselves, are all interrelated. The framework is flexible and can accommodate unique investment objectives and constraints corresponding to different types of investors and situations.

Importantly, the framework allows investors to measure the cost of liquidity – "what is the cost of making my portfolio more liquid?". Increasing a portfolio's liquidity – or, increasing the confidence of meeting all cash flow obligations – typically implies that the portfolio must become less risky and have a lower allocation to private assets. Investors may find that a high liquidity requirement can be particularly expensive in terms of the expected portfolio performance. By quantifying the cost of liquidity, some investors may conclude that their portfolios may be too liquid.

Acknowledgement

We wish to thank Thomas Barton, Dr. Taimur Hyat, Cameron Lochhead and Michael Samaha for their valuable contributions to this project.

Appendix

A1. LP Investment Definitions[21]

LP buyout funds: This is a broad private equity fund LP investment. These funds include "equity investments in mature companies that result in a change of control. These are typically larger transactions that use leverage. Relatively narrower range of returns can be expected."

LP mezzanine funds: This is a private mezzanine debt fund LP investment. These funds include "debt investments which are subordinate to other debt in the capital structure and are backed by little to no collateral. Securities are generally term loans and notes; may contain warrants/conversion rights."[22]

LP real estate funds: This is an equity, "value added" real estate investment. These funds include "investments in properties that require moderate to significant capital expenditures for renovations and/or the solving of various operational, management or capital constraint issues. Occupancy rates are expected to be moderate to high. Return is expected to be driven primarily by appreciation."

A2. Asset Liquidation Selling Rule

With the recognition of the lumpiness and higher transaction costs of private assets relative to public assets, and the acknowledgement that private assets' main function in the asset allocation framework is not as a source of ready cash, we specify the following "selling rule" to guide the simulation on which of the five assets to sell, and in which order, to generate cash:

a. First, use any contributions and income. If insufficient, then

b. Sell liquid public assets, *pro rata.* If insufficient, then

c. Sell LP allocations, *pro rata*. If it is the first occurrence to liquidate private assets, 50% of LP allocations is sold, *pro rata,* and the proceeds after transaction costs are reinvested *pro rata* in the public portfolio. If it is the second occurrence of the need to sell private assets, all remaining LP allocations are sold, and the proceeds after transaction costs are reinvested *pro rata* in the public portfolio. This selling rule tries to capture that LP investors are reluctant to sell and exit private investments due to reputational effects and getting shut out of future private investment opportunities. In addition, we impose a lock-up period of two years from the outset, assuming it is a good economy period. If the LP allocation value is insufficient, or if liquidation of the LP allocation must occur during the lock-up period, then

d. Declare that the particular simulation run has "failed" to meet the investor's liquidity requirement.

The framework can accommodate other selling rules.[23]

21 Descriptions of the three LP asset classes are from Burgiss: *Private Capital Classification System*, page 3, 2017. For this study, we rely on Burgiss for LP investment performance data.

22 These debt LP investments are not "direct lending/middle market" debt LP investments.

23 Alternative LP selling rules are possible, for example, selling lower transaction cost LP investment first, etc. However, the selling rule used in the Case Study tries to maintain asset allocation weights within both the public and private portfolios.

A3. Summary of Case Study Assumptions

Figure A1: Summary of Asset Allocation Case Study Input Assumptions

Public Assets – Investor Inputs		
	Liquid Low-Risk (5-10y IG Corporate Index)	Liquid High-Risk (S&P 500 Index)
	Return Statistics	
Annualized Expected Return	3.60%	8.00%
Annualized Standard Deviation	5.30%	14.10%
	Correlation	
Liquid Low-Risk	1	–
Liquid High-Risk	0.27	1

Private Assets – Investor Inputs			
	LP Investment Type		
	LP Buyout	LP Mezz. Debt	LP Real Estate
Allow? (YES/NO)	YES	YES	YES
Income (%/y)		6%	4%
	LP Transaction Costs		
"Good" Economy	5%	5%	5%
"Bad" Economy	30%	15%	9%
	LP Capital Call Assumptions		
Stub Period (years)	2	0	1
% Never Called	0%	0%	0%
Default Investment	Public assets portfolio	Public assets portfolio	Public assets portfolio
	Investor Views on Expected Future LP Performance		
Quartile 1 (Q1 – highest)	25%	25%	25%
Q2	25%	25%	25%
Q3	25%	25%	25%
Quartile 4 (Q4 – lowest)	25%	25%	25%
	Investor Fund-Selection Skill		
Quartile 1 (Q1 – highest)	25%	25%	25%
Q2	25%	25%	25%
Q3	25%	25%	25%
Quartile 4 (Q4 – lowest)	25%	25%	25%
	LP Diversification Parameters		
Number of LP Funds	5	5	5
Number of LP Vintage Years	1	1	1

Note: Provided for illustrative purposes only.

A4. PME (Public Market Equivalent)

The performance metric to derive private LP investment values is the private market equivalent (PME) ratio. The PME measures the relative performance of a private market investment to that of a public market index. LP investments are characterized by cash outflows and cash inflows which are not under the investor's control. The cash outflows are the contributions made by an investor to the LP fund over a number of years as the GP makes capital calls. The cash inflows are the distributions back to the investor as the GP liquidates fund holdings and distributes the proceeds.

Instead of investing in the private LP fund an investor could decide to invest the contributions in the public market (*i.e.*, the benchmark) following the same exact capital call schedule. The PME is the ratio of what the investor earned from the private investment relative to what they would have earned in the public market. To allow for a suitable comparison, distributions from the fund are reinvested in the public market.

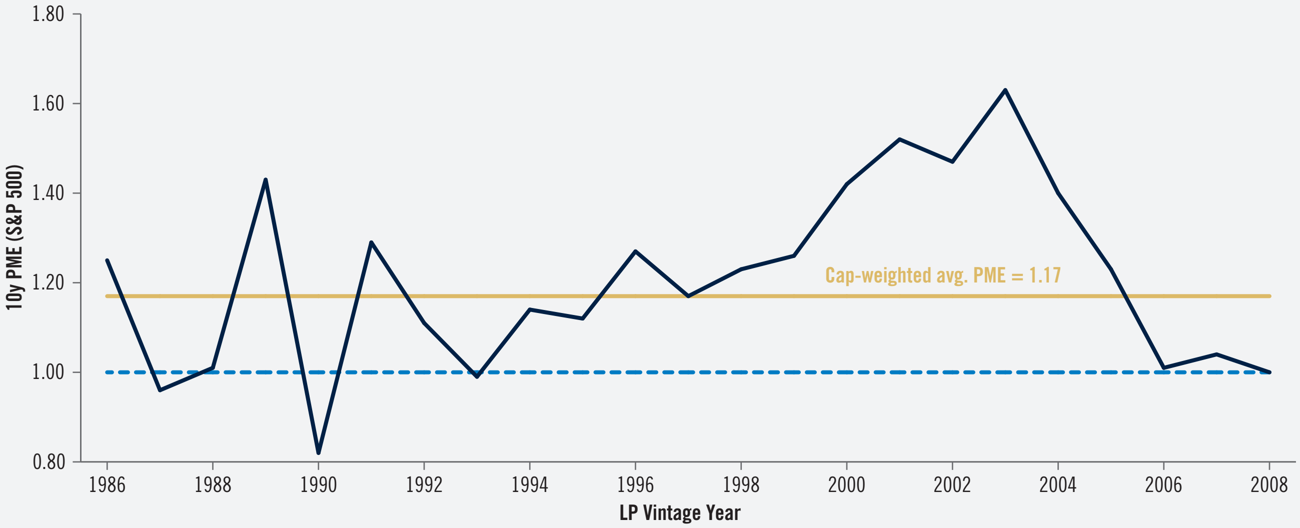
Presented as a formula, the time T PME is the reinvested value of all distributions brought forward to time T divided by the invested value of all calls also brought forward to time T:

$$PME(T) = \frac{FV(Distributions)}{FV(Calls)}$$
$$FV(Distributions) = \sum_{s}^{T} Distribution(s) \times (1 + TR_{bmk}(s,T))^{T-s} \qquad \text{(A1)}$$
$$FV(Calls) = \sum_{t}^{T} Call(t) \times (1 + TR_{bmk}(t,T))^{T-t}$$

$TR_{bmk}(s,T)$ is the annualized total return of the public benchmark portfolio from time s to time T.

Figure A2 shows the capital-weighted 10y PMEs for US LP buyout funds by vintage year from 1986 to 2008. The public market index used in the PME calculation is the S&P 500 Index. A PME greater than one implies that the investor would have been better off by contributing capital to the LP investment compared to investing in the public market. For example, a 10y PME value of 1.2 means that the investor would have made an additional 20%, including public reinvestment, at the end of the 10y investment horizon compared to a strategy of investing the contributions in the public benchmark. Over the 23y period the capital-weighted average PME is 1.17, indicating that LP buyout funds have, on average, outperformed the S&P 500 Index.

Figure A2: LP Buyout Funds 10y Pooled Vintage-Level PMEs (SPX)



Source: Burgiss, PGIM IAS. Provided for illustrative purposes only.

A5. Call Schedule: Logistic Function Parameter Fitting

Figure A3: Logistic Function Parameter Estimates

Dependent variable:	$Drawdown^{B/O}$ coeff.	$Drawdown^{Mezz.}$ coeff.	$Drawdown^{RE}$ coeff.
a_0	0.057	0.107	0.118
a_1	0.071	0.053	0.233
a_2	0.094	–0.004	–0.190
b_0	0.967	0.806	1.176
b_1	–0.278	–0.050	–0.507
b_2	–0.235	0.169	1.167
# observations	528	475	410
Vintages	*1986–2017*	*1986–2017*	*1987–2017*

Source: Burgiss, PGIM IAS. Provided for illustrative purposes only.

A6. Estimating the PME – Public Market Relationship

The initial PME estimate is built on a regression model where horizon (10y) pooled vintage-level PMEs are regressed on 10y annualized total returns of public equity (SPX) and public US High Yield debt (US HY). We do this separately for LP buyout ($PME_v^{B/O}$), LP mezzanine debt (PME_v^{Mezz}) and LP real estate (PME_v^{RE}). For the latter two we find that the SPX alone provides a good fit.

$$10y\ PME_v^{B/O\wedge} = \alpha(10y) + \beta_{SPX}(10y) \times TR_{SPX}(0,10y) + \beta_{HY}(10y) \times TR_{HY}(0,10y) + \varepsilon(10y)$$

$$10y\ PME_v^{Mezz.} = \alpha(10y) + \beta_{SPX}(10y) \times TR_{SPX}(0,10y) + \varepsilon(10y)$$

$$10y\ PME_v^{RE.} = \alpha(10y) + \beta_{SPX}(10y) \times TR_{SPX}(0,10y) + \varepsilon(10y)$$

$$TR_{SPX}(0,10y) = \left(\frac{SPX_{t=10}}{SPX_{t=0}}\right)^{\frac{1}{10}} - 1$$

$$TR_{HY}(0,10y) = \left(\frac{HY_{t=10}}{HY_{t=0}}\right)^{\frac{1}{10}} - 1 \qquad \text{(A2)}$$

Figure A4 shows the regression results.

Figure A4: 10y Pooled Vintage-Level PME Estimation Results

Dependent variable:	10y $PME_v^{B/O}$		10y $PME_v^{Mezzanine}$		10y PME_v^{RE}	
	coeff.	*t-stat*	*coeff.*	*t-stat*	*coeff.*	*t-stat*
constant	1.09	9.78	1.34	44.95	1.41	6.39
10y TR^{SPX}	–3.77	–6.22	–3.31	–3.74	–4.7	–1.9
10y TR^{HY}	5.5	3.07				
# observations	23		10		11	
R^2	53%		42%		36%	
SER	0.16		0.17		0.40	
Vintages	*1986–2008*		*1998–2007; exclude 1999*		*1997–2009; exclude 2000–01*	

Note: Newey-West standard errors. Several OLS independent variables combinations have been tried for the buyout, mezzanine and real estate regressions, including 10y S&P 500 Index Total Return only, 10y S&P Index Total Return and 10y US HY Total Return, and 10y S&P 500 Index Total Return and 10y US IG Total Return.

Source: Burgiss, PGIM IAS. Provided for illustrative purposes only.

A7. Calculating the LP Investment Value

This example illustrates how the LP investment horizon value is calculated.

Required inputs are the total commitment amount, the capital call schedule, annual total return of the benchmark public asset and the estimated interim PMEs. Table A5 has our input assumptions and resulting LP investment values.

We consider a 10y investment horizon, $100 of committed capital and the S&P 500 as the benchmark. Column *a* has a cumulative capital call schedule over the 10y horizon. Yearly contributions in column *b* are calculated based off of the call schedule and the total commitment amount. An example of the S&P 500 annual total return is provided in column *c*. In column *d* we have the interim PME values which are estimated solely from an empirical linear relationship using public equity (S&P 500) and public high yield debt (Bloomberg Barclays US HY Index) total returns. Sampling from a PME prediction error distribution (as described in Section "Estimating vintage-level PMEs") is skipped in this example. For the LP buyout interim PME at time *n* we have

$$n_{year}\ PME_v^{B/O\wedge} = \alpha(n_{year}) + \beta_1(n_{year}) \times TR_{SPX}(0,n_{year}) + \beta_2(n_{year}) \times TR_{HY}(0,n_{year}) \quad \text{(A3)}$$

LP investment values are the product of PMEs and the benchmark asset value when invested in accordance to the LP call schedule. Mathematically, the interim benchmark asset values, assuming that the contributions/capital calls are invested in an S&P 500 strategy are:

$$\textit{Benchmark value}_i = (\textit{Benchmark value}_{i-1} + \textit{Contribution}_i) \times (1 + r_i)$$

$$\textit{Benchmark value}_0 = 0$$

$\textit{Contribution}_i$: *call amount at the beginning of year i (column b)*

r_i: *Benchmark asset(S&P 500) annual return of year i (column c)* (A4)

Column *e* contains the interim and horizon value of capital calls invested in the benchmark asset. At the beginning of year 1, $20 is called. If the $20 is invested in the S&P 500, with an annual return of 10%, the end-of-year benchmark value will be $22 (= 20 + 0.1 × 20). At the beginning of year 2, another $10 is called, the total $32 will grow at a rate of 8% and become $34.56 (= (22 + 10) × (1 + 0.08)). Repeating the calculations for each subsequent period yields a horizon value of $180.8.

To get the LP investment value (column f), we multiply the benchmark value with the estimated PME.

Figure A5: LP Investment Value Calculation Example

	a	b	c	d	e	f (=d×e)	g	h (=f+g)
Year	Call schedule (Assumed)	Yearly Contribution at the beginning of the year ($)	S&P 500 return (simulated) (r_i)	Interim PME (estimated)	Benchmark Value ($) (assuming calls invested in S&P 500)	LP investment value ($)	Earnings on investing uncalled capital in S&P 500 ($)	LP allocation value ($)
1	20%	20	10%	0.90	22.0	19.8	88.0	107.8
2	30%	10	8%	1.01	34.6	34.9	84.2	119.1
3	50%	20	12%	1.03	61.1	62.9	71.9	134.9
4	60%	10	5%	1.04	74.7	77.3	65.0	142.3
5	65%	5	7%	1.07	85.2	91.2	64.2	155.5
6	80%	15	2%	1.09	102.2	111.4	50.2	161.7
7	85%	5	5%	1.10	112.6	123.9	47.5	171.4
8	100%	15	10%	1.13	140.4	158.6	35.7	194.4
9	100%	0	12%	1.16	157.2	182.4	40.0	222.4
10	100%	0	15%	1.17	180.8	211.5	46.0	257.6

Source: PGIM IAS. Example shown for illustrative purposes only.

A8. Calculating the LP Allocation Value

The LP allocation value incorporates the earnings of the uncalled capital amount. Column *h* of Table A5 shows the LP allocation as the sum of the LP investment value and the earnings of the uncalled amount (column g). For ease of illustration we have assumed that the uncalled capital is invested in the S&P 500. The invested value of the uncalled capital can be calculated as:

$$\text{Earnings of uncalled capital}_i = (\text{Earnings of uncalled capital}_{i-1} - \text{Contribution}_i) \times (1 + r_i)$$

$$\text{Earnings of uncalled capital}_0 = \text{Initial total commitment}$$

Contribution_i: *contribution amount at the beginning of year i (column b)*

r_i: *Benchmark asset(S&P 500) annual return of year i (column c)* (A5)

In Figure A5, column *g* has the value of the invested uncalled capital at each time step. At the beginning of year 1, $20 is called by the LP fund resulting in $80 of uncalled capital. Investing the uncalled capital in the S&P 500 results in an end-of-year value of $88 (= 80 × (1 + 10%)). At the beginning of year 2, another $10 is called, leaving $78 of uncalled capital which will grow a rate of 8% in year 2. Proceeding in this manner results in $46 of earnings due to investing of yet-to-be-called capital. The LP horizon allocation value equals $257.6, the sum of LP horizon investment value ($211.5) and $46 earnings from investing uncalled capital in the S&P 500.

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