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WHAT IS THE OPTIMAL NUMBER OF EQUITY MANAGERS? A CIO TOOLKIT FOR MANAGER ALLOCATION

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We quantitatively answer the question "What is the optimal number of managers when forming a multi-manager portfolio?" We develop an optimization tool (MAP) that constructs a diversified multi-manger portfolio. MAP allows a CIO to express preferences across different manager types while also incorporating manager selection skill, management fee function and operation cost.

We apply our portfolio construction toolkit to US large-cap equity manager allocation. Given a particular input of portfolio settings, we solve for the optimal number of managers and associated manager allocations that can achieve the highest expected information ratio. We also find that the optimal number of managers increases, from 3-4 to 8-9 managers, as the CIO's manager selection skill improves from low skill to high skill.

While MAP addresses the active management portion of the whole portfolio, we illustrate how a CIO can use this toolkit along with passive strategies to construct optimal portfolios that seek to obtain a specified tracking error volatility target.

Many institutional investors have adopted a multi-manager structure to achieve desired equity exposures and meet return objectives. The goal is to harvest alpha from various sources efficiently with diversified active risk. However, CIOs must decide the number of managers to hire and the corresponding allocation across different kinds of managers.

Researchers and practitioners have long studied an analogous problem in stock portfolio construction. A US large-cap equity portfolio of approximately 30 stocks, without significant sector bias, is diversified enough to exhibit overall volatility only marginally different from the benchmark index (Statman, 1987). However, this result cannot be simply applied in the context of constructing a multi-manager portfolio. The performance dispersion among long-only US large-cap equity active managers is much lower than that among stocks, reducing the number of managers needed to control overall portfolio active risk. In addition, diversifying across managers is more expensive than across stocks. Since the management fee rate typically rises with lower allocations, a portfolio consisting of many managers with a small allocation to each would lead to overall higher cost, especially for investors with limited total capital. In addition, CIOs incur internal costs of manager search, due diligence and periodic supervision. Such costs rise, perhaps rapidly, as CIOs employ more managers.

While the literature on modern portfolio diversification extends back to the 1950s, less has been written in the field of manager portfolio construction. One of the earliest articles on this topic was O'Neal (1997) who finds that combining a few randomly picked managers shrinks the dispersion of terminal wealth and the total return series volatility converges to the level of benchmark volatility. More recently, Garvey, Kahn and Savi (GKS, 2017) discuss the diversification effect on different components of manager active returns. They treat active manager performance as a package of "generic" and "idiosyncratic" ideas and show that a combination of randomly picked active managers leads to a portfolio where generic ideas (*e.g.,* market and other systematic risk factors) dominate the portfolio's risk budget while exposure to uncorrelated ideas (*e.g.,* manager idiosyncratic ideas) shrinks. The diversification benefit vanishes when the market declines significantly and correlations among generic ideas spike. GKS' proposed solution, in the context of a multi-strategy fund, is to have the fund focus on idiosyncratic ideas and increase that risk without using leverage. However, this would be a task for the fund manager and require sophisticated security-level management of each underlying strategy. A CIO facing the multi-manager portfolio construction challenge cannot easily implement this solution as the CIO does not control the strategies underlying the funds.

To assist the CIO, we develop a manager allocation methodology to help solve the problem of achieving an efficient portfolio active risk-return trade-off. The associated optimal number of managers should balance the benefits and costs of diversification.

We focus on the US large-cap equity investment mandate and identify three persistent manager attributes: **style, investment approach** and **active risk level**. Managers' characteristics over these three dimensions identify distinct investment philosophies and strategies, thereby partitioning the manager universe. Based on this classification, we develop an allocation tool to help CIOs make manager allocation decisions. This tool allows CIOs to customize total investment capital, management fee structure, investment operation cost function, preferences on different types of managers, manager selection skills, etc. We use simulation to determine the optimal total number of managers to hire given the CIO's cost function, manager preferences and manager selection skill.

We proceed as follows: After describing the data we identify three manager characteristics to categorize managers into peer groups. We then introduce the **Manager Allocation Programming (MAP)** tool which identifies manager allocations that match CIO preferences. Finally, given the CIO's manager selection skill level, we simulate the investment experience based on monthly manager performance data from January 1997 to December 2017, considering management fees and operation costs, to determine the optimal number of managers.

Data

Using the eVestment Alliance database, we collect information on US large-cap equity managers who reported performance anytime between January 1997 and December 2017 with a continuous performance reporting length of at least five years. For each manager, we record the manager's style, investment approach and monthly performance (gross) data. The data are self-reported.

Each US large-cap manager belongs to one style sub-universe: Core, Value or Growth. The Russell 1000 index and its corresponding value and growth style tilts are the sub-universe benchmarks used to measure a manager's active performance. Investment approach describes how the manager forms investment views, either by quantitative models or fundamental analyses.[1] Since the cessation of a manager's monthly performance reporting is usually associated with a material business event (*e.g.,* merger and acquisition, business discontinuation, or major fund liquidation), we assume report discontinuation signals the manager's business termination.

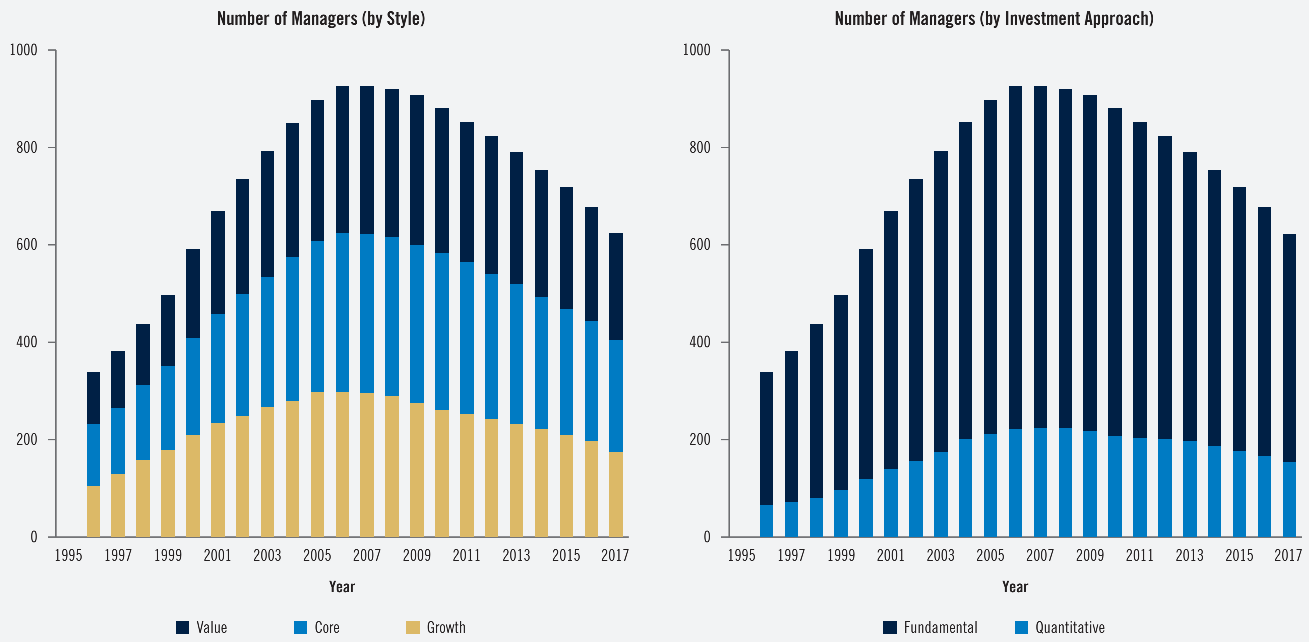
Figure 1 shows the number of managers actively reporting performance, by year, style and investment approach.

The total number of active managers rose from less than 400 to more than 900 before the financial crisis, trending down to around 600 by the end of 2017. Most of this reduction is accounted by the fundamental manager group whose count has shrunk by more than 250 from the peak.

In our attempt to avoid survivorship bias, we consider all managers (both "active" and "dead") recorded in the eVestment database who reported anytime during the 21y period from January 1997 to December 2017.

1 For each manager, the entry of investment approach can be "quantitative", "fundamental" or "not reported" ("NA"). We exclude the minor number of NA managers.

Figure 1: Number of Active US Large-cap Equity Managers



Source: eVestment, PGIM IAS. Provided for illustrative purposes only.

Manager Characteristics

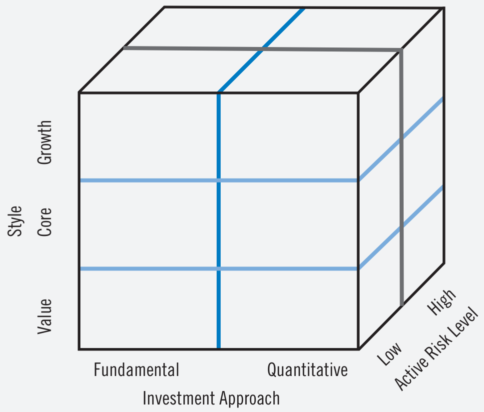
GKS points out a potential problem when CIOs combine managers: Less risk budget will be allocated to idiosyncratic ideas when investment capital is distributed across more managers. This danger of overdiversification highlights the importance of an efficient manager allocation methodology. The key to successful multi-manager portfolio construction is *to diversify across distinct investment ideas, not managers*. While market exposure is common among almost all long-only equity managers, some other ideas, not necessarily perfectly idiosyncratic, are common only within a certain manager type. For example, many quantitative funds may develop factor premium strategies which are based on similar analytics and logics, producing similar long-term performance. However, fundamental managers are unlikely to use such strategies. Similarly, a growth-stock pitch may be popular among the cohort of growth equity managers but is unlikely to gain interest from value managers. Therefore, the multi-manager portfolio structure can be efficient if it is composed of fundamentally distinct strategies while controlling for the dilution of manager-specific risk.

We propose three manager characteristics to categorize US large-cap equity managers: **style**, **investment approach** and **active risk level**.

Managers declare their investment style as either Core, Growth or Value. The manager's style tends to greatly influence active performance as it determines both the manager's opportunity set and performance benchmark.[2] We also record a manager's investment approach. We focus on managers who adopt either fundamental or quantitative analyses to form investment decisions. Facing the universe of stocks defined by the benchmark, fundamental managers form views on stocks' forward performance by analyzing companies' financial reporting, liaising with management and making discretionary judgements on industry trends. Depending on the tracking error volatility constraint and other manager criteria, a fundamental manager's portfolio with a US large-cap mandate may have as few as 20-30 stocks. In contrast, a typical quantitative manager has well-defined rules, which is why quantitative strategies are sometimes called "systematic." While strategies of different quantitative managers can vary in detail, most of them, if not all, are derived from and tested using the historical data set. Typically, a quantitative manager's portfolio has hundreds of stocks.

2 We measure manager performance by the Russell 1000 index for managers in US large-cap core sub-universe, the Russell 1000 Growth index for managers in US large-cap growth sub-universe, and the Russell 1000 Value index for managers in the US large-cap value sub-universe.

Figure 2: Partitioning the US Large-cap Equity Manager Universe



Source: PGIM IAS. Provided for illustrative purposes only.

The third manager characteristic, active risk, measures the average variability of a manager's performance relative to the benchmark over time. Managers holding a portfolio of a few stocks with large active weights tend to show high active risk, while managers with low active risk structure portfolios to constrain *ex ante* active risk. We define a manager's active risk level—either low or high—relative to the active risk level of the manager's peer group.[3]

In summary, we partition the universe of US large-cap equity managers into twelve blocks (Figure 2). The cube sorts managers by differences in investment ideas. Analysis of variance (ANOVA), which tests whether the groups are statistically distinct, helps validate this partitioning. We run ANOVA on managers' alpha and tracking error volatility. Figure 3 summarizes the ANOVA results for manager active performance. Managers characterized by distinct style and active risk level exhibit different alpha and tracking error volatility, on average, as evidenced by the low p-value. Note that investment approach does not indicate significant differences in alpha but does partition by active risk. The unequal active return and risk behaviors across manager groups suggest our partitioning of managers does segregate investment ideas.

The advantage of this partitioning is its stability as it is based on slow-moving manager characteristics. Other attempts to identify the structure of the manager universe includes measuring managers' exposures. Using regression analysis, the total return of a manager can be attributed to benchmark and systematic factor (*e.g.,* Fama-French factors) exposures plus a residual. However, the lack of statistical significance and robustness, especially for systematic factor beta, makes this approach challenging. Another concern is that the active exposures estimated by time series regression fail to capture ideas that are tactical but nonetheless common among certain groups of managers. Finally, the reliability of time series regression is undermined as manager performance data are not aligned since some managers have performance records for the full period while others start only after 2000 and then stop with relatively short performance series.

We acknowledge that there might be other manager characteristics that may better categorize distinct strategies compared to our partitioning. However, our analytics framework can incorporate other dimensions.

Figure 3: Identifying Manager Characteristics
ANOVA (One-Way) of Style, Investment Approach and Active Risk Level

	F-statistics & p-value	Style	Investment Approach	Active Risk Level
Alpha	F-statistic	12.4	1.4	75.0
	p-value	0.0%	23.8%	0.0%
Tracking Error Volatility	F-statistic	51.5	38.0	502.5
	p-value	0.0%	0.0%	0.0%

Note: The classification considers all managers that have reported performance from January 1997 to December 2017 with a performance time series length of at least five years. Provided for illustrative purposes only.
Source: eVestment, PGIM IAS.

3 Given a group of managers sharing the same style and investment approach (*e.g.,* growth and fundamental managers), we decompose the group into sub-groups with high/low active risk level relative to each other. Although a manager's active risk level relative to peers is not available from the database, we infer managers' active risk level by examining their active returns. CIOs can infer a manager's active risk level, though subject to short-term shocks, by manager interview. See Appendix for details.

Manager Allocation Programming (MAP)

We split the problem of finding the optimal number of managers into two parts: First, *given a desired total number of managers, what is the set of possible manager allocations, from the universe of managers, that best fit a CIO's preferences?* Then, assuming the CIO selects from this efficient set of manager allocations, *what is that total number of managers that achieves the best risk-adjusted active performance?* We develop the MAP (Manager Allocation Programming) tool to solve the first part with an additional assumption that the CIO distributes the capital evenly across all managers. We discuss the second part of the problem in a later section.

MAP solves for the best allocations across manager types using constrained optimization, allowing the CIO to express preferences to certain types of managers (see Appendix for MAP details). CIOs may have preferences to certain types of investment ideas and may wish to allocate more capital to managers that implement these ideas. For example, a CIO who has deferred liability payments may seek long-term returns and wish to tilt towards managers with high conviction, concentrated portfolios with high active risk. Alternatively, a CIO who believes that growth-focused managers are more skilled may wish to tilt to this manager style.

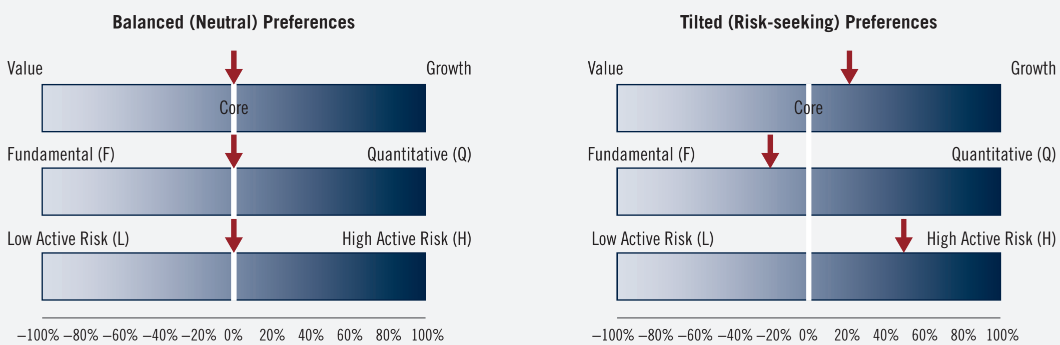
Given a specified total number of managers and the assumption of equal capital distribution, MAP searches for manager allocations that minimize the gap between the CIO's desired and realized manager preferences. To illustrate how MAP accommodates a CIO's preferences we consider two examples (Figure 4):

- **Balanced (Neutral)**: The CIO has no explicit preference to any manager type and is comfortable with a portfolio balanced across various investment ideas.
- **Tilted (Risk-seeking)**: The CIO, with a preference for fundamental managers, seeks long-term returns and tolerates short term volatility. The CIO prefers 20% (net) of capital managed by growth managers (*i.e.* 20% more of the CIO's capital is allocated to growth managers than value managers); 20% (net) of capital managed by fundamental managers; and 50% (net) of capital managed by high active risk managers.[4]

One important feature of MAP is that it discriminates across deviations in the three manager characteristics. In other words, the three characteristics are not equal in their ability to identify distinct investment ideas (recall Figure 3). The investment approach only identifies differences in tracking error volatility (TEV), while the other two dimensions lead to distinct outcomes for both alpha and TEV. This implies that a deviation over the investment approach characteristic would have less impact on the CIO's experience and therefore MAP gives it less weight compared to the other two dimensions. We configure MAP to match a CIO's manager preferences with decreasing priority across style, active risk level and investment approach. A CIO can alter this hierarchy, if desired, by changing the relative scale of the deviation penalty multipliers.

While MAP allocates across managers to reflect a CIO's preferences as accurately as possible, MAP imposes a small concentration penalty to favor allocations involving more manager types. For example, in the case of a total of three managers with Neutral CIO preferences, MAP favors an allocation with one manager from each of the core, value and growth groups, instead of an allocation that has three core managers.

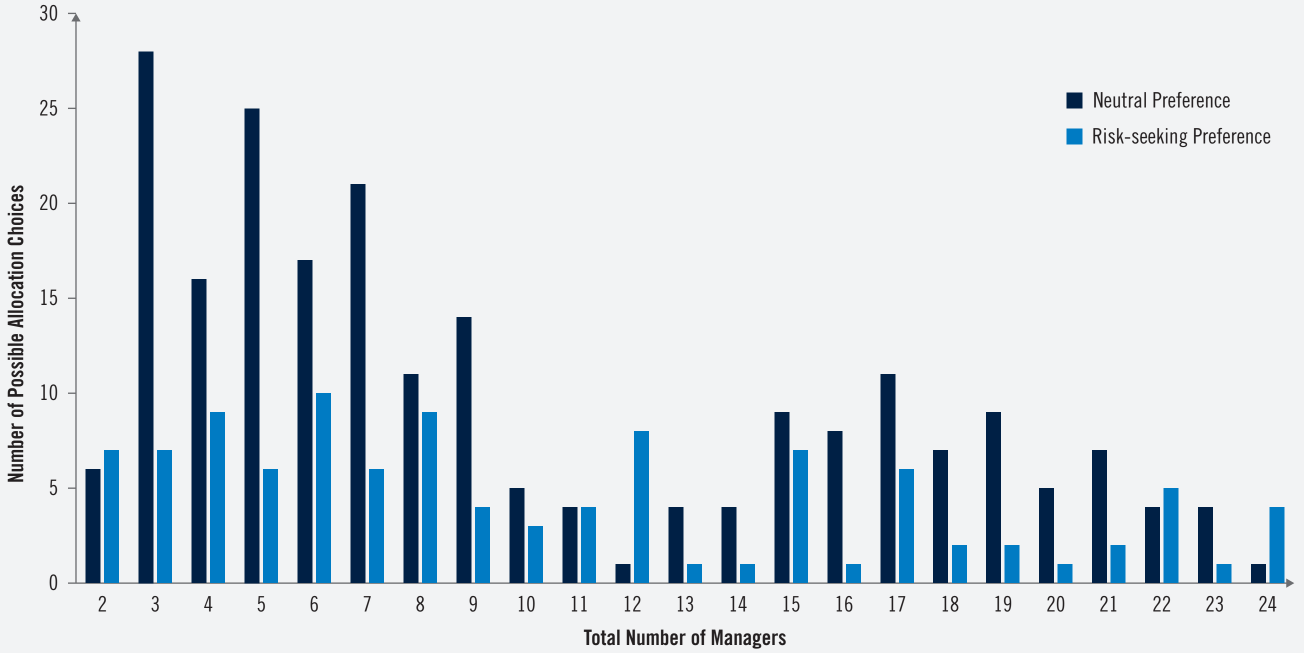
Figure 4: CIO Manager Preferences – Two Examples



Source: PGIM IAS. Provided for illustrative purposes only.

4 Over the style dimension, core is a balanced mix of value and growth. A CIO can either express view in net terms which sets a relatively loose constraint and allows the core style to act as a buffer, or specify an exact percentage of capital to be managed by each manager type. See Appendix for details.

Figure 5: Number of Possible MAP Allocations that Best Fit CIO Preferences



Note: We assume the CIO evenly distributes capital across managers and sets the priority hierarchy to match the CIO's preferences across style, investment approach and active risk level. The concentration penalty is active.

Source: PGIM IAS. Provided for illustrative purposes only.

Given a CIO's total number of managers, MAP identifies all possible manager allocation choices that best fit the CIO's preferences (Figure 5). Since the Risk-seeking preferences impose stronger constraints on possible manager allocations compared to the Neutral preferences, MAP identifies fewer best-fit allocation choices.

As an example, Figure 6 shows the allocations that best match Neutral and Risk-seeking preferences. Given a total of ten managers, MAP finds five possible best-fit allocations for Neutral preferences and three allocations for Risk-seeking preferences. Figure 6 shows each of the ten managers' classification by style (Core, Value, and Growth), investment approach (F and Q), and active risk level (L and H). From the perspective of the style dimension, MAP allocations include both a "barbell" type (Allocation #1) and a "core-satellite" type (Allocations #2 – #5 for Neutral preferences and Allocations #2 – #3 for Risk-seeking preferences), as neither of the two preference types directly constrain the allocation to core. A CIO chooses the specific manager allocation from MAP's best-fit allocation solutions.

Modeling Cost and CIO Manager Selection Skill

Given a total number of managers MAP identifies the possible best-fit manager allocations to match a CIO's preferences. But what is the optimal total number of managers? Since the total number of managers employed affects management fees and operation cost, and, hence, overall portfolio performance, we must account for these factors before identifying the optimal number of managers. In addition, a CIO's skill in identifying good performing managers is also likely to affect the optimal number of managers – good selection skill allows a CIO to avoid some of the performance cost of adding managers to improve portfolio diversification.

Management Fees

Management fees are typically proportional to the size of the investment. The market convention is to charge a management fee based on the amount of capital invested, sorted into tranches.[5] The fee rate is highest for the bottom tranche, and falls as the invested capital touches higher tranches, up to a ceiling. The existence of such "fee breaks" makes it cost-efficient, all else equal, to hire fewer managers.

5 We apply the same analysis to the case of performance-based management fee. The result is available upon request.

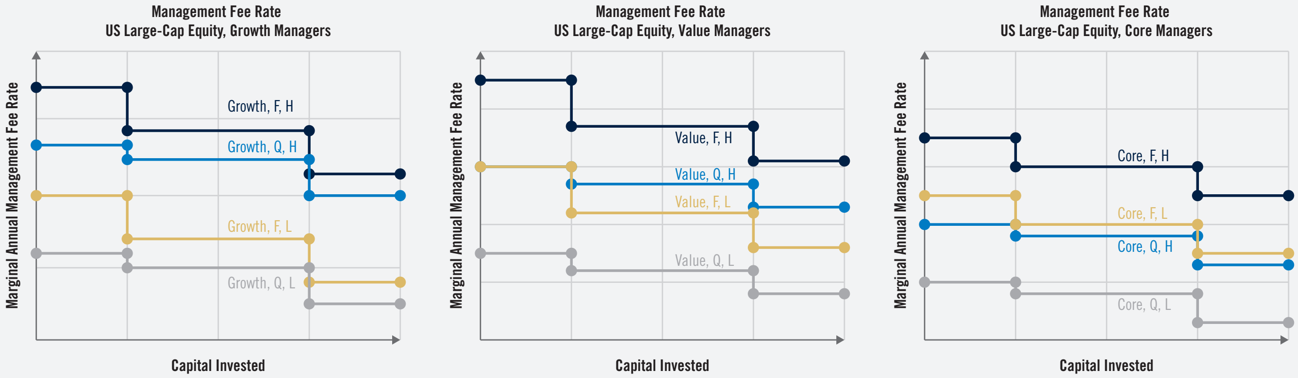
Figure 6: MAP Solutions for a 10-Manager Allocation

		Balanced (Neutral) Preferences					Tilted (Risk-seeking) Preferences		
		Allocation # 1	Allocation #2	Allocation #3	Allocation #4	Allocation #5	Allocation #1	Allocation #2	Allocation #3
Manager 1		Value,F,L	Value,F,L	Value,F,L	Value,F,L	Value,F,H	Value,F,L	Value,F,H	Value,F,H
Manager 2		Value,F,H	Value,F,H	Value,F,H	Value,Q,L	Value,Q,L	Value,F,H	Value,Q,H	Value,Q,H
Manager 3		Value,Q,L	Value,Q,L	Value,Q,H	Value,Q,H	Value,Q,H	Value,Q,H	Core,F,L	Core,F,L
Manager 4		Value,Q,H	Core,F,L	Core,F,L	Core,F,L	Core,F,L	Core,F,H	Core,F,H	Core,F,H
Manager 5		Core,F,L	Core,F,H	Core,F,H	Core,F,H	Core,F,H	Core,Q,H	Core,F,H	Core,Q,L
Manager 6		Core,Q,H	Core,Q,L	Core,Q,L	Core,Q,L	Core,Q,L	Growth,F,L	Core,Q,H	Core,Q,H
Manager 7		Growth,F,L	Core,Q,H	Core,Q,H	Core,Q,H	Core,Q,H	Growth,F,H	Growth,F,L	Growth,F,L
Manager 8		Growth,F,H	Growth,F,H	Growth,F,L	Growth,F,L	Growth,F,L	Growth,F,H	Growth,F,H	Growth,F,H
Manager 9		Growth,Q,L	Growth,Q,L	Growth,Q,L	Growth,F,H	Growth,F,H	Growth,Q,L	Growth,Q,L	Growth,F,H
Manager 10		Growth,Q,H	Growth,Q,H	Growth,Q,H	Growth,Q,H	Growth,Q,L	Growth,Q,H	Growth,Q,H	Growth,Q,H
Portfolio Exposure	Manager Style	0	0	0	0	0	0. 2	0. 2	0. 2
	Investment Approach	0	0	0	0	0	-0. 2	-0. 2	-0. 2
	Active Risk Level	0	0	0	0	0	0. 4	0. 4	0. 4

Note: We assume the capital is evenly distributed across the ten managers at the beginning of investment period with no active rebalancing. Portfolio exposure matches CIO's preference in Figure 4. Provided for illustrative purposes only.

Source: PGIM IAS.

Figure 7: Hypothetical US Large-Cap Equity Active Management Fee Rate



Source: PGIM IAS. Provided for illustrative purposes only.

Figure 7 illustrates a hypothetical annual management fee structure used to simulate net-of-fee performance. The management fee rate varies across strategies. Value/growth-focused managers tend to charge more than core managers, and fundamental (F) managers tend to cost more than quantitative (Q) managers. High (H) active risk portfolios concentrating on few names with high conviction and expected to realize success over the long term, are usually more expensive compared to portfolios with low (L) active risk. CIOs can apply their own assumptions.

Operation Cost

Operation cost is another important factor that influences the optimal number of managers. Manager sourcing, due diligence and surveillance involve a CIO's time and energy. A CIO must set up trust accounts, conduct regular interviews and monitor the employed managers, including any change in management structure, technology improvements, evolution of investment methodologies, etc. These costs grow (probably at an increasing rate) as more managers are hired, making it costly to hold a portfolio with numerous managers.

We model operation cost as increasing non-linearly with the number of managers. Figure 8 shows a hypothetical internal operational cost structure where 10 managers produce an overall annual portfolio cost of 10bp/y, and 20 managers cost 40bp/y. The accelerating cost reflects higher operational risk of getting just one "bad apple" manager in a basket of managers which can call into question the CIO's overall portfolio management, diminishing the appeal of hiring a lot of managers. The multiplier (η) reflects a CIO's governance efficiency and can be set by the CIO.[6] A smaller multiplier corresponds to a better ability to manage a given number of managers.

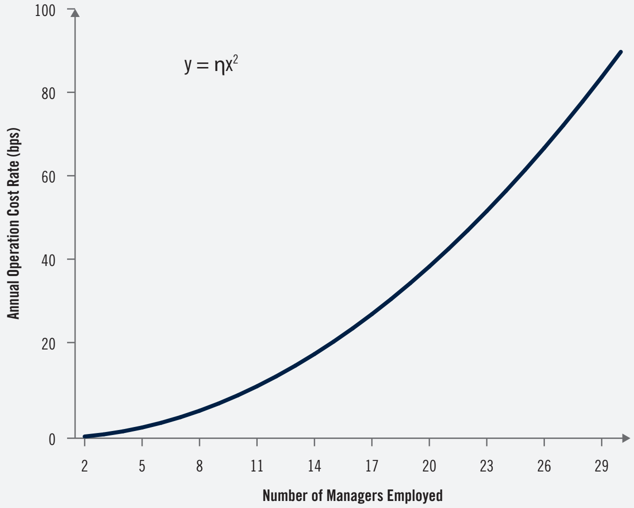
Manager Selection Skill

Just as portfolio managers research companies to identify potential winners and losers, a CIO adopting a multi-manager portfolio structure conducts due diligence and manager interviews to evaluate a manager's alpha generation ability. These efforts should improve the CIO's chances to do better than just randomly selecting managers.

We model the CIO's manager selection skill utilizing the probability distribution function of a selected manager's forward performance rank to peers. As the CIO's manager selection skill improves, the selected managers will more likely have above-average forward performance. A CIO with no selection skill randomly picks managers. So the probability of a selected manager's forward performance rank relative to peers can range equally anywhere from worst to best (*i.e.* a uniform distribution). In contrast, a CIO with selection skill is more likely to hire a manager with performance better than the peer average. Therefore, the probability that a selected manager will outperform the median manager will likely be greater than 50%. Figure 9 shows relative performance rank distribution of a selected manager corresponding to three levels of CIO manager selection skill.

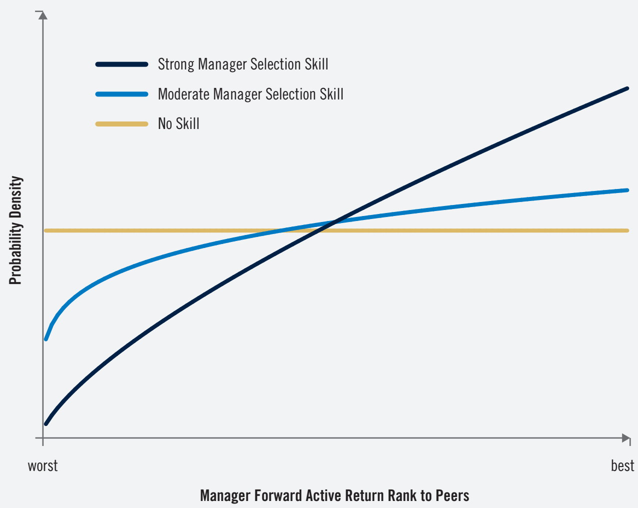
The manager selection skill is reflected by the correlation of selection probability and realized performance ranking to peers. A CIO with no selection skill has an equal chance (50% probability) of hiring a portfolio manager with an alpha either above or below the median of its peers in the following investment period. In contrast, a CIO with moderate manager selection skill has a 56% probability of picking a manager with an alpha higher than the group median. This probability rises to 69% if the CIO possesses strong selection skill. We can, in practice, map the corresponding manager selection skill level with the selection probability for a manager by quartile (Figure A3).

Figure 8: Hypothetical Annual Operation Cost Rate of a Multi-Manager Portfolio



Source: PGIM IAS. Provided for illustrative purposes only.

Figure 9: Probability Density Function of Manager's Forward Performance Rank to Peers, by CIO Skill Level



Source: PGIM IAS. Provided for illustrative purposes only.

6 We set $\eta = 0.1$. As currently specified, a portfolio of ten managers will incur an operation cost of 10bp each year.

Notably, our model of CIO selection skill produces an unbalanced effect on the tails of the manager selection probability distribution. A skillful CIO can effectively eliminate the possibility of selecting a manager who significantly underperforms but can only moderately increase the chance of partnering with the best. We adopt this nonlinear and conservative assumption since it may be easier to eliminate the bottom performers rather than select the top stars. By interviewing the portfolio managers, a skillful CIO can avoid managers with flawed decision-making processes, weak risk management and obsolete technology, but is less able to predict the best performers. (See Appendix for details.)

Finding the Optimal Number of Managers

MAP solves the first part of the optimal number of managers problem by identifying the best-fit allocations that match a CIO's preferences. Once the best-fit allocations are identified, the optimal total number of managers is determined by multiple additional inputs, including but not limited to, the total amount of capital to invest, management fee and operation cost functions, and the CIO's manager selection skill. Given these inputs, we simulate the investment experience using actual manager performance from January 1997 to December 2017 – a long period across many market environments – to identify the optimal total number of managers.

To illustrate, we assume the CIO has $1b to invest in the US large-cap equity market and faces management fee and operation cost as specified above. To better evaluate MAP's performance compared to other manager allocation methodologies, we consider the "Core-only" method (*i.e.,* pick managers only from the core group) since it is a natural choice for CIOs who measure performance by the core benchmark. In addition, we also consider the "Core-satellite (style)" method (*i.e.,* pick managers from the core, value and growth group), motivated by the favorable result as discussed in Amenc, Malaise and Martellini (2004).

We simulate and compare the investment experiences for a CIO in a variety of settings: Neutral/Risk-seeking preferences, either zero/moderate/strong selection skill and three possible manager allocation methods (MAP, Core-only and Core-satellite (style)).

CIOs often evaluate manager performance every 3-5y. Therefore, we divide the 21y data period into seven, non-overlapping 3y periods. In each period, the CIO selects managers to form a multi-manager portfolio. For each 3y period we calculate portfolio alpha (net of management fees and operation cost), tracking error volatility (TEV) and the *ex post* information ratio (IR). We average these performance metrics over all seven periods to estimate the *expected* 3y performance metric and determine the optimal number of managers relying on the expected portfolio information ratios.[7] (See Appendix for simulation details.)

Balanced (Neutral) Preferences

We first consider a CIO with Neutral preferences and zero manager selection skill. We examine investment performance using the three different manager allocation methodologies:

- **Core-only:** The CIO picks managers only from the core group;
- **Core-satellite (style):** The CIO adopts a core-satellite structure along the style dimension, evenly distributing capital across core, value and growth managers.[8] The CIO does not control the portfolio's exposure over manager's investment approach and active risk level; and
- **MAP:** The CIO allocates managers following the MAP solutions.

Figures 10 and 11 summarize the results. The management fee shifts down the portfolio alpha evenly across different total number of managers. The operation cost grows nonlinearly (by construction) with the total number of managers, especially after more than 10 managers are hired.

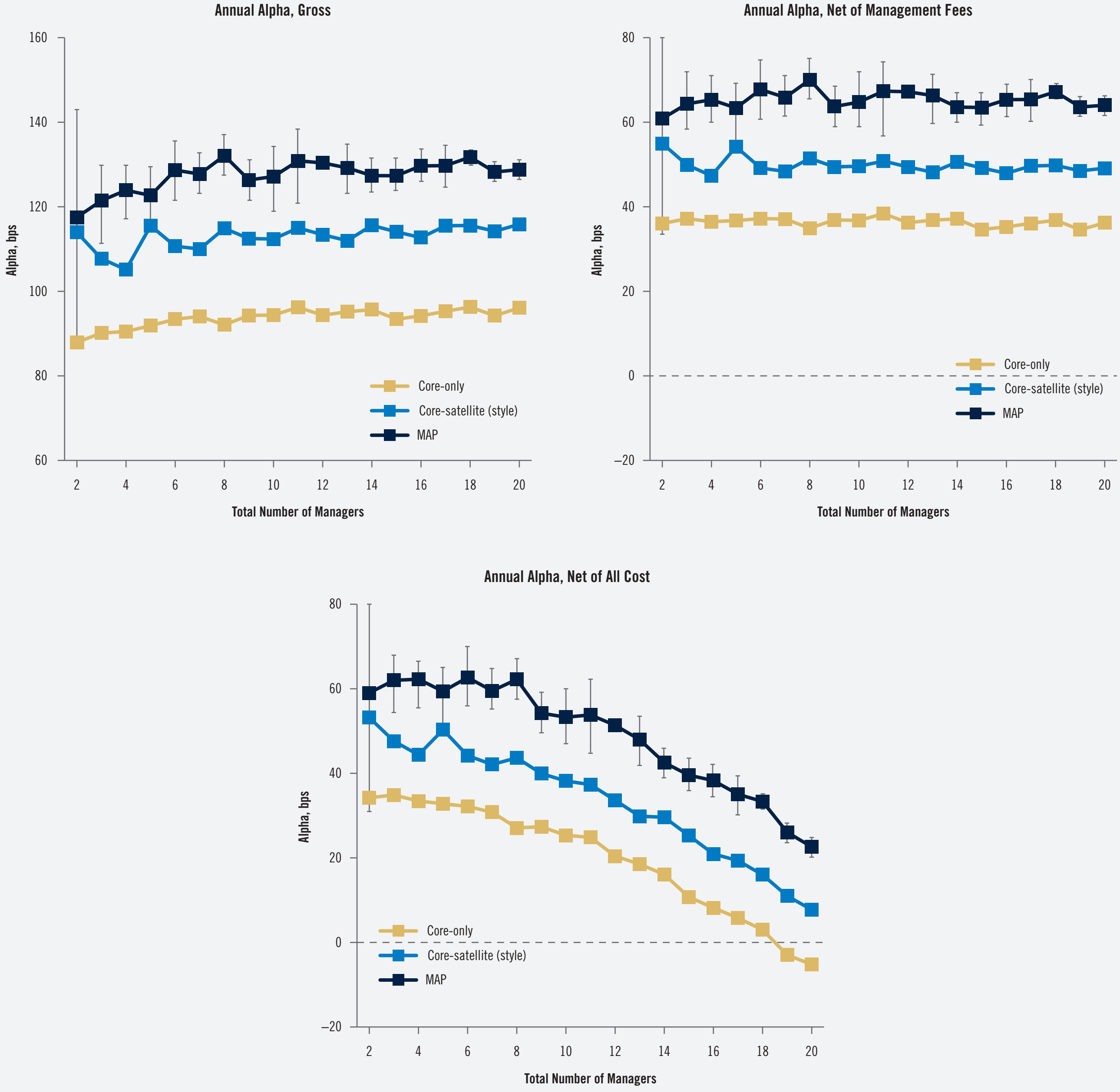
The Core-satellite (style) method outperforms Core-only, in terms of alpha (gross and net), for any total numbers of managers. In addition, although the Core-satellite (style) portfolio has higher TEV, the portfolio average 3y IR is always better (Figure 11). This result implies that the Core-satellite (style) structure guides CIO to allocate managers more efficiently and take on more "good" volatility.

The MAP allocation method dominates both the Core-only and Core-satellite (style) methods. MAP allocates managers across manager characteristics (Figure 2) while maintaining portfolio net exposure close to, if not exactly, neutral over all three dimensions. While all three methods lead to portfolios with net neutral (core) portfolio exposure (Figure 12), only MAP recognizes and adjusts for the unintended bias to the fundamental investment approach and low active risk level produced by the other two methods (therefore slightly increasing the portfolio's tracking error volatility). Compared to the Core-satellite (style) structure, MAP solutions further optimize risk allocations and therefore produce both higher average annual IR and net alpha, by 0.02 – 0.10 and 10 – 30bp/y, respectively, depending on the total number of managers.

7 The portfolio achieving the highest information ratio is also the portfolio that adds the most value to investors. Grinold (1989) shows the value added by active management is proportional to the squared portfolio information ratio.

8 Extra weight, if any, will be allocated to the core group. For example, with a budget of 10 managers, the CIO will hire 3 managers in value/ growth group and 4 managers in core group.

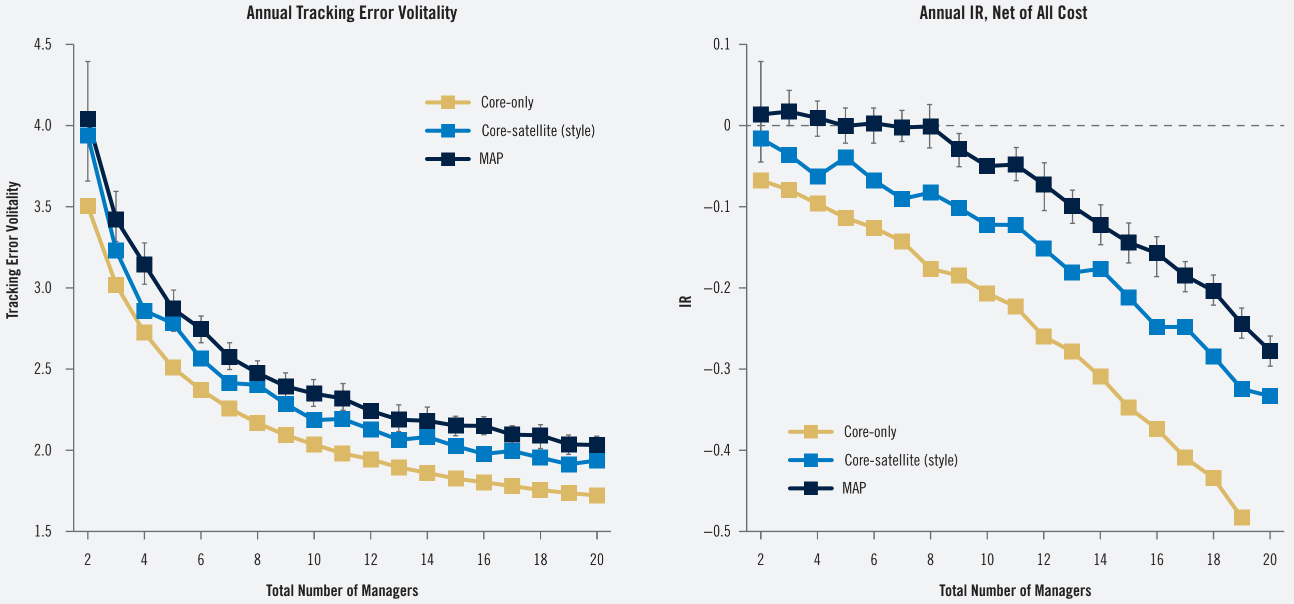
Figure 10: Simulated Average Portfolio 3y Annual Alpha, by Manager Allocation Method
(neutral preferences; no manager selection skill)



Note: The simulation is based on historical manager performance data from January 1997 to December 2017. We show the average result of the seven, non-overlapping 3-year periods. For a given total number of managers, both Core-only and Core-satellite (style) methods produce one manager allocation solution, while MAP has multiple solutions. The error bars of the MAP solutions represent 95% confidence intervals of the average over all solutions. A narrow confidence interval indicates similar performance across MAP allocations. The CIO is assumed to have zero manager selection skill.

Source: eVestment, PGIM IAS. Provided for illustrative purposes only.

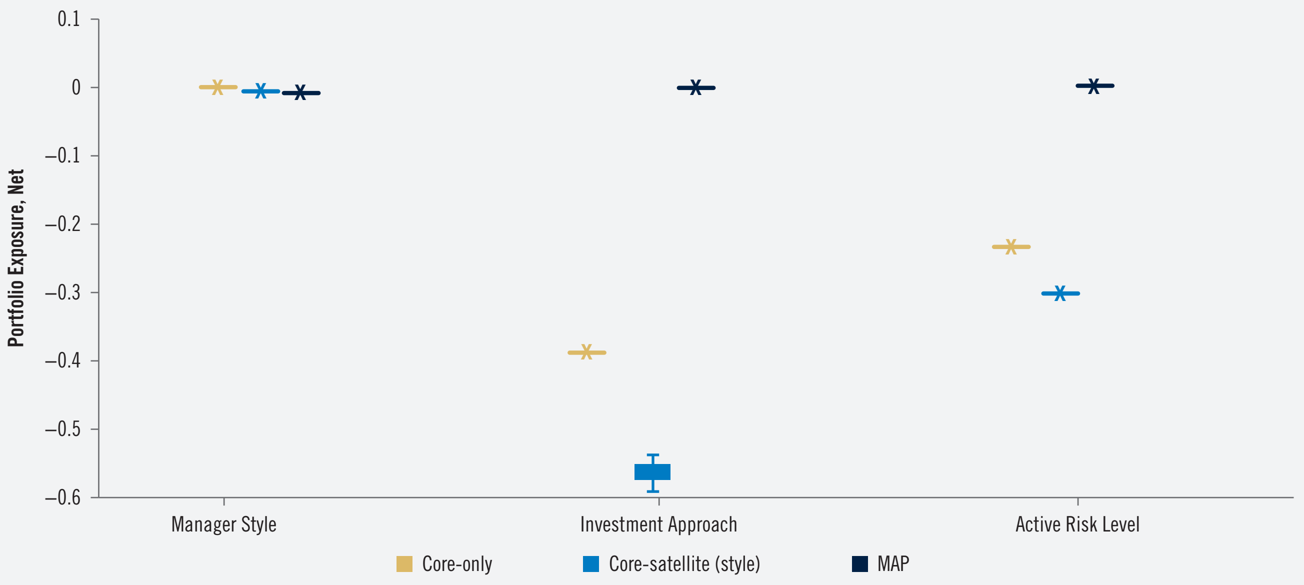
Figure 11: Simulated Average Portfolio 3y Annual TEV and IR, by Manager Allocation Method
(neutral preferences; no manager selection skill)



Note: The simulation is based on historical manager performance data from January 1997 to December 2017. We show the average result of seven, non-overlapping 3-year periods. For a given total number of managers, both Core-only and Core-satellite (style) correspond with one manager allocation solution, while MAP has multiple solutions. The error bar of MAP solutions represents a 95% confidence interval of the average over all solutions. A narrow confidence interval indicates similar performance across MAP allocations. The CIO is assumed to have zero manager selection skill.

Source: eVestment, PGIM IAS. Provided for illustrative purposes only.

Figure 12: Distribution of 3y Average Portfolio Exposure to Manager Characteristics, by Manager Allocation Method



Note: The result is shown across different total number of managers. Average is marked by cross. The error bars represent the first and third quartiles. Provided for illustrative purposes only.

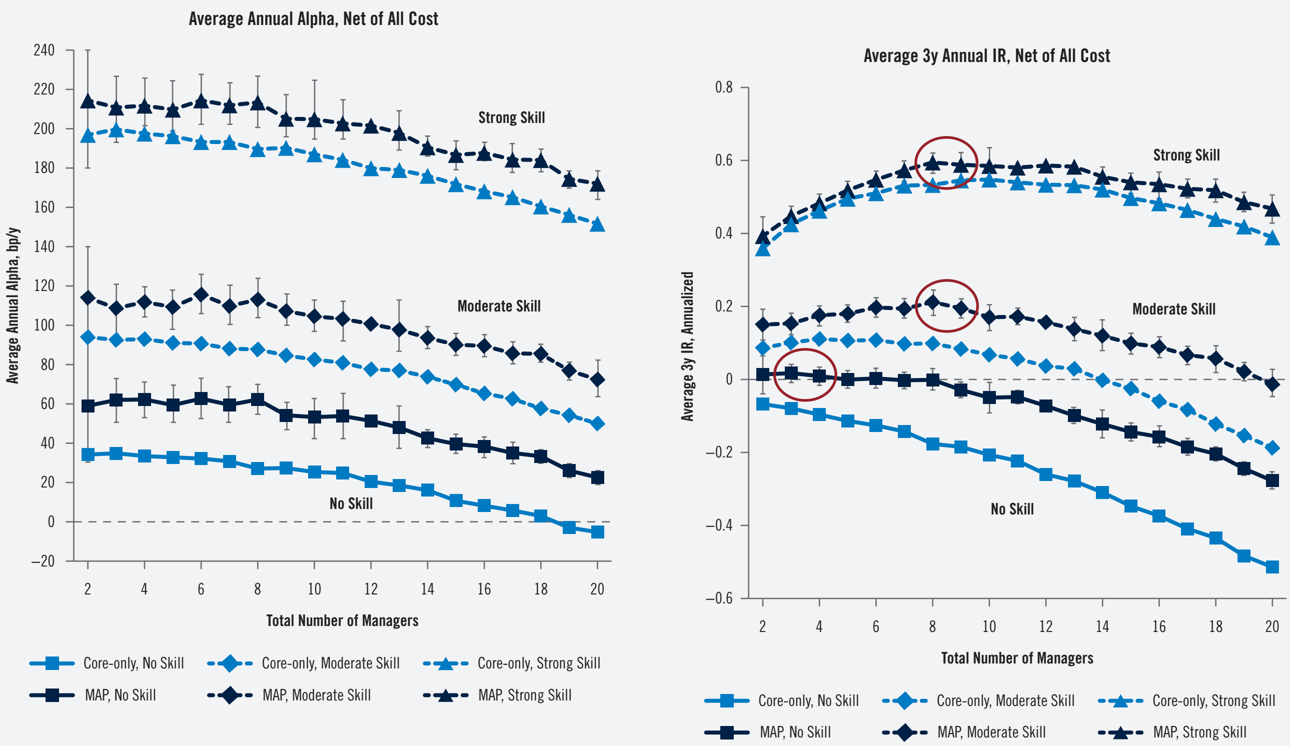
Source: eVestment, PGIM IAS.

Note that for all three allocation methods the annualized IRs averaged over the seven, non-overlapping 3y periods are barely positive (Figure 11), while the net alphas with fewer than five managers range from 30 to 60bp/y (Figure 10). This is due to the observed positive correlation between a manager's alpha and TEV. Negative alphas are often associated with relatively low TEVs which have an effect of increasing the IR (negative) magnitude. In contrast, positive alphas are often associated with relatively high TEVs which tend to decrease the IR (positive) magnitude. Overall, 3y periods with a negative alpha have a relatively larger impact on the average IR compared to 3y periods with large positive alphas. As a result, the distribution of *ex post* IR exhibits a negative skew even if alpha is normally distributed, producing the observed disconnect between the average alpha and average IR.[9]

MAP allocations perform better than Core-only and Core-satellite (style) due to its control not only over style but also across the other manager characteristics (*i.e.*, investment approach and active risk level). In fact, the Core-satellite (style) structure is a subset of MAP solutions. However, unlike the Core-satellite (style) method, MAP rules out portfolio constructions that are unbalanced with respect to the other manager characteristics, providing a more efficient risk-return tradeoff. This explains why MAP solutions exhibit more attractive average IRs despite having slightly higher tracking error volatility compared to Core-satellite (style) solutions. We focus on Core-only and MAP allocations in the remaining discussion.

Figure 13 shows the effect of CIO manager selection skill. Not surprisingly, the average annual alpha shifts up as CIO manager selection skill improves. While MAP continues to dominate Core-only, in terms of both net alpha and information ratio, the spread narrows as skill improves. The value added of adopting MAP instead of Core-only allocation method peaks when the CIO has little manager selection skill. Importantly, irrespective of the manager selection skill level, MAP will likely perform at least as well as Core-only.

Figure 13: Simulated Average Portfolio 3y Alpha and IR
(neutral preferences; different CIO manager selection skill levels)



Note: The simulation is based on historical manager performance data from January 1997 to December 2017. We show the average result of seven, non-overlapping 3y periods. For a given total number of managers, the Core-only method produces one manager allocation solution while MAP has multiple solutions. The error bar of MAP solutions represents a 95% confidence interval of the average over all solutions. A narrow confidence interval indicates similar performance across MAP solutions. The information ratios, achieved by the optimal number of managers following the MAP method with each of the three levels of manager selection skill, are marked by red circles. Provided for illustrative purposes only.

Source: eVestment, PGIM IAS.

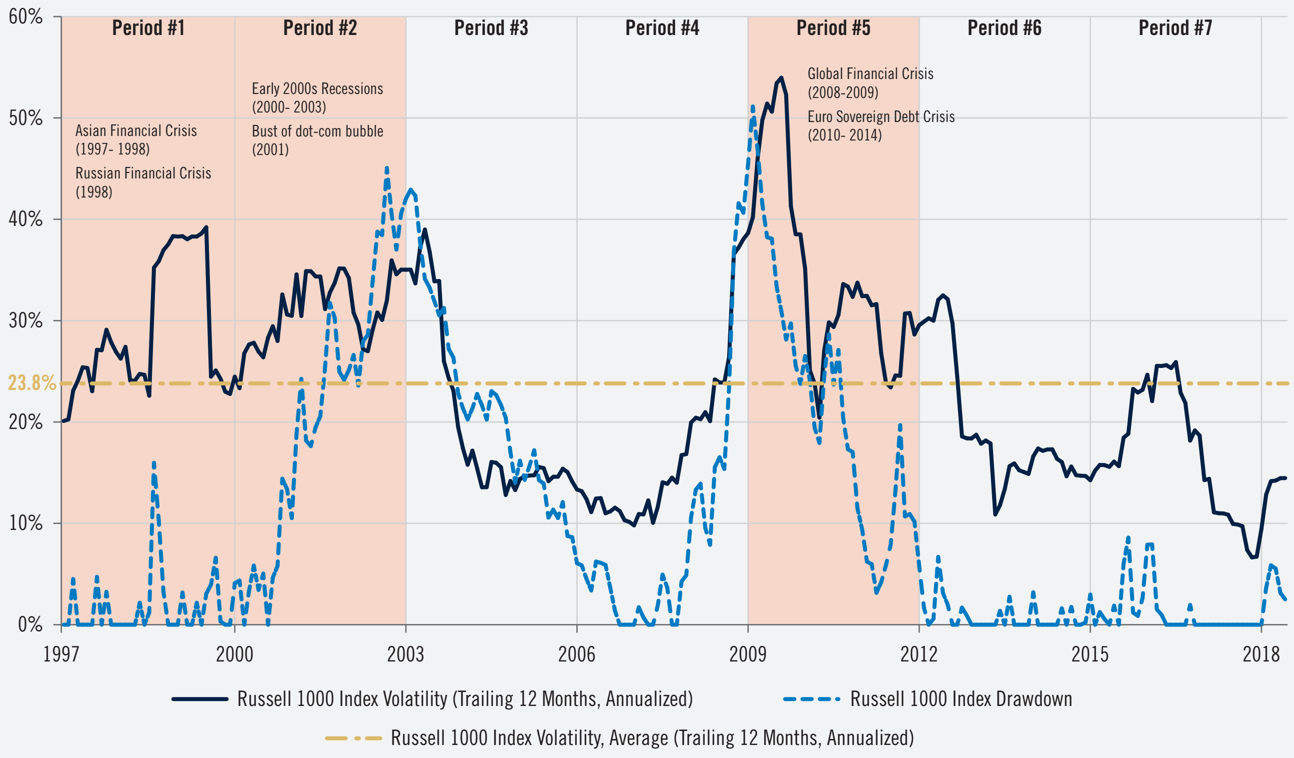
9 The mismatch of *ex post* average alpha and average information ratio can also be explained as follow: E[IR] = E[alpha/TEV] is not equal to E[alpha] × E[1/TEV] as alpha and TEV are (positively) correlated.

As the number of managers increases, average net alpha at first remains flat and then declines quickly as operation costs escalate. The dispersion of net alpha achieved by different MAP allocations (represented by the error bars in Figure 13), given the same total number of managers, narrows as the CIO adds more managers, implying increasingly similar investment results from different MAP allocations. The average 3y IR shows a humped pattern, especially for a skillful CIO. At first the IR rises as the number of managers increases, but then declines when more managers are hired. The initial IR rise is due to the initial rapid decline in TEV when allocating to more than two managers. However, as more and more managers are hired the net alpha decay and the diminishing marginal diversification benefit causes the average IR to start falling.

The hump average IR pattern is less pronounced if the CIO has little manager selection skill as the portfolio TEV quickly levels off (see Appendix for details). This is because the average active performance correlation among the "above average cohort", who are more likely to be selected by a skillful CIO, is higher than that of the whole group. In contrast to a CIO with no skill, it takes more managers for a skillful CIO to fully exploit manager diversification, until increasing cost of portfolio management drives down net alpha.

What is the optimal number of managers? The answer depends on the *expected* portfolio information ratio which we estimate from the simulation average. We have shown that MAP produces more efficient manager allocations while accurately reflecting the CIO's preferences. Introducing more managers reduces TEV and therefore amplifies the information ratio by a multiplier. A CIO without skill achieves an *ex post* average IR across all seven non-overlapping 3-year periods not much different from zero and there is no significant change in the information ratio as the number of managers increases. However, for a more skillful CIO, the *ex post* average IR starts positive so the improvement in the information ratio as the tracking error volatility falls is more noticeable until the alpha decay drives it down. Considering the portfolio information ratio (and net alpha), all else equal, we find that a CIO with Neutral preferences and following MAP allocations benefits from an increasing number of managers as selection skill improves: 3-4 managers in the case of no skill, and 8-9 managers in the case of moderate to strong skill.[10]

Figure 14: Russell 1000 Total Return Volatility (Trailing 12m, Annualized) and Drawdown



Note: We estimate the 12m trailing annual volatility and running drawdown based on Russell 1000 index monthly returns. We identify three periods characterized by higher than long-term average volatility (shaded). Provided for illustrative purposes only.

Source: Datastream, PGIM IAS.

10 A similar result was reported in a hedge fund investment study (Wintner, Zaslavskiy, and Abdulali (2014)).

Manager Allocation Performance during Stressful Market Environments

As GKS highlights, the main drawback of a traditional multi-manager construction is the illusion of low risk. Despite having many managers there may be few ideas driving active performance. We believe correlations between these ideas are low when the market is behaving normally, but spike during market turmoil which often occurs with changed market dynamics and poor manager performance. In other words, the diversification effect vanishes when it is most needed.

To address this critique, we examine how MAP allocations perform during stressful 3y market environments. To locate challenging 3y market periods, we use volatility as an identifier. We find three periods (Figure 14) characterized by higher than average market volatility: January 1997 – December 1999; January 2000 – December 2002; and January 2009 – December 2011. Each 3y period is associated with some crisis event(s). Some are local to US financial markets while others are global but spill over to the US.

Figure 15 compares the simulated performance in each of the seven 3y periods between Core-only and MAP allocations assuming Neutral preferences and moderate manager selection skill. Each method has its optimal total number of managers (4 for Core-only and 8 for MAP; see Figure 13). At first glance, the average portfolio active performance during the three challenging 3y periods (Periods #1, #2 and #5) is even better than the overall average. However, this is due to the strong outperformance over January 2000 – December 2002 (Period #2) when both allocation methodologies earned more than 600bp/y average annualized alpha. For a conservative analysis, we exclude Period #2. For the Core-only allocation, the average annual alpha during Period #1 and #5 falls to -50bp/y compared to the long-term average 93bp/y, a drop of 143bp/y. MAP allocations generated 20bp/y versus 113bp/y, a smaller 93bp/y drop. The MAP method, though not immune from the problem of correlation spikes, has improved the portfolio's performance in high volatility regimes by more effective manager diversification.

MAP dominates Core-only from the perspective of both average net alpha and IR in most periods. The exception is Period #4 (January 2006 – December 2008) due to the different exposure along the investment approach dimension. While MAP allocations maintain balanced exposure over all three manager characteristics, the Core-only allocation approach leads to an (unintended) overweight to fundamental managers (Figure 12). This bias, combined with the quantitative fund meltdown in August 2007 (Amir and Lo, 2011), creates an advantage against neutral investment approach allocations. This observation supports our treatment of portfolio exposure to manager characteristics as risk factors that should be managed. CIOs can tilt to any of these characteristics for either tactical or strategic preferences but the key to long-term success, given a constant level of manager selection skill, is an efficiently diversified portfolio construction that achieves the desired portfolio exposure.

Figure 15: Average Net Alpha, Tracking Error Volatility (TEV) and IR of Each 3y Period

(neutral preferences; moderate CIO manager selection skill)

	Core-only Allocations, 4 Managers			MAP Allocations, 8 Managers		
3y Investment Period	Average Alpha (bp/y), Net of All Cost	Average TEV	Average IR, Net of All Cost	Average Alpha (bp/y), Net of All Cost	Average TEV	Average IR, Net of All Cost
#1 1997 Jan -1999 Dec	-24	3.26%	-0.09	56	3.32%	0.05
#2 2000 Jan- 2002 Dec	608	5.55%	1.06	605	4.99%	1.23
#3 2003 Jan- 2005 Dec	91	2.05%	0.34	196	2.09%	0.88
#4 2006 Jan- 2008 Dec	190	2.52%	0.75	61	2.27%	0.28
#5 2009 Jan- 2011 Dec	-76	2.55%	-0.36	-16	2.16%	-0.10
#6 2012 Jan- 2014 Dec	-67	1.64%	-0.47	-36	1.39%	-0.33
#7 2015 Jan- 2017 Dec	-71	1.73%	-0.45	-73	1.45%	-0.53
Average of All Periods	**93**	**2.76%**	**0.11**	**113**	**2.52%**	**0.21**
Average of Periods #1, #2, #5	**169**	**3.79%**	**0.20**	**215**	**3.49%**	**0.39**
Average of All Periods (ex. #2)	**7**	**2.29%**	**-0.05**	**31**	**2.11%**	**0.04**
Average of Periods #1, #5	**-50**	**2.91%**	**-0.22**	**20**	**2.74%**	**-0.03**

Note: We calculate the net alpha, tracking error volatility and net information ratio for each of the seven nonoverlapping 3y periods. The result is averaged across simulation paths. The three challenging periods (shaded) featured by high volatility. Provided for illustrative purposes only.

Source: eVestment, PGIM IAS.

Figure 15 also shows that manager active performance is not stationary. Investment performance during January 2000 – December 2002 (Period #2) was a big outlier and increases the long-term average. However, the Appendix shows that the main results hold excluding this period as the shape of curves (Figure 13) is roughly maintained. This implies that the outliers mainly have a level effect. From the perspective of a CIO, over changing market environments, MAP solves for manager allocations that achieve an efficient active risk-return trade-off and the optimal number of managers.

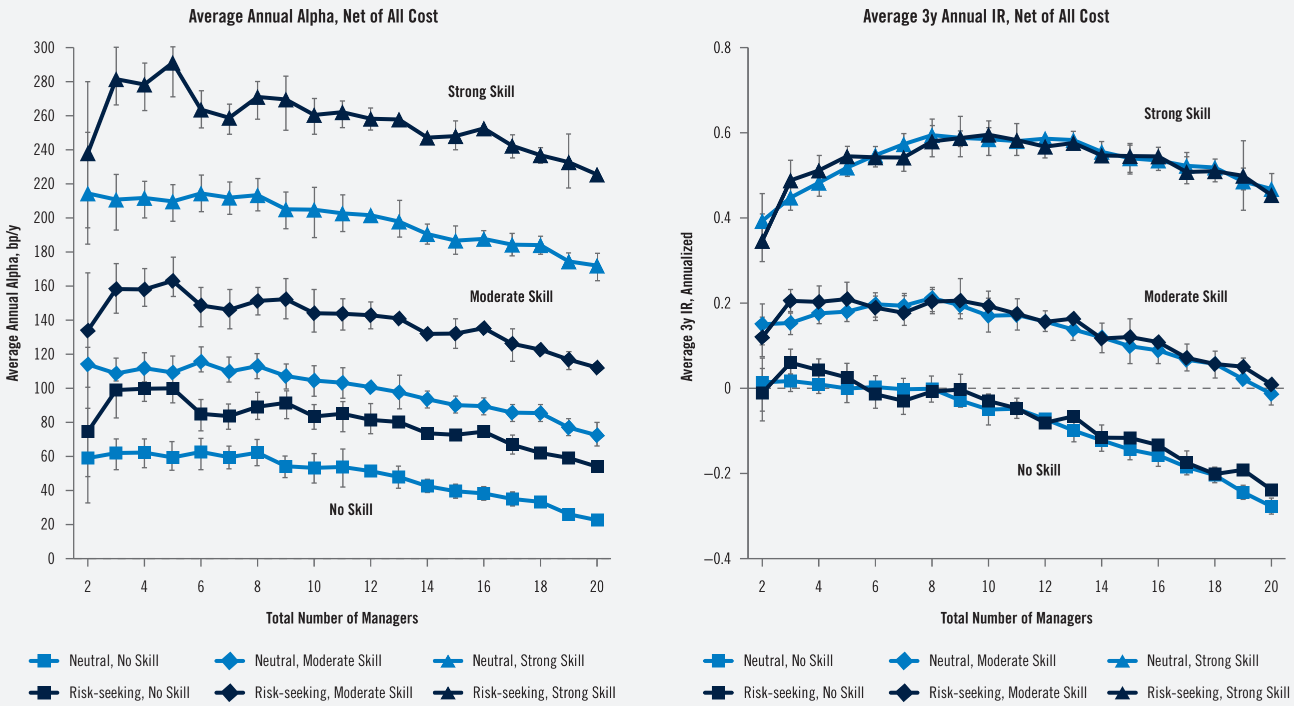
Tilted (Risk-seeking) Preferences

Attracted by the strong post-2008 performance of growth stocks relative to value, some CIOs may consider deviating from core with a secular overweight to growth (McElhaney, 2017). How have MAP allocations with Risk-seeking preferences performed?

We assume Risk-seeking preferences have a net 20% tilt to growth (Figure 4). To appropriately measure active performance, we customize a benchmark by combining Russell 1000 Value and Russell 1000 Growth indices to produce the same tilt (see Appendix for details). Figure 16 shows the simulated investment experience of MAP allocations, with different levels of manager selection skill, under Neutral and Risk-seeking preferences.

The curves of net alpha versus the total number of managers shift up as the CIO's manager selection skill improves. All else equal, Risk-seeking portfolio dominates the Neutral portfolio in terms of average net alpha. The curves of the Risk-seeking portfolio are a little bumpy when there are few managers as the preferences are more challenging to match with fewer managers (recall that we assume the capital is equally distributed among managers). The average IR curves generated by different preferences surprisingly overlay each other within the range of errors. We find that the Risk-seeking portfolio boosts active returns by assuming more risk (Figure 17). The MAP method maintains the same efficiency of active risk-return trade-off irrespective of our sets of CIO's preferences.

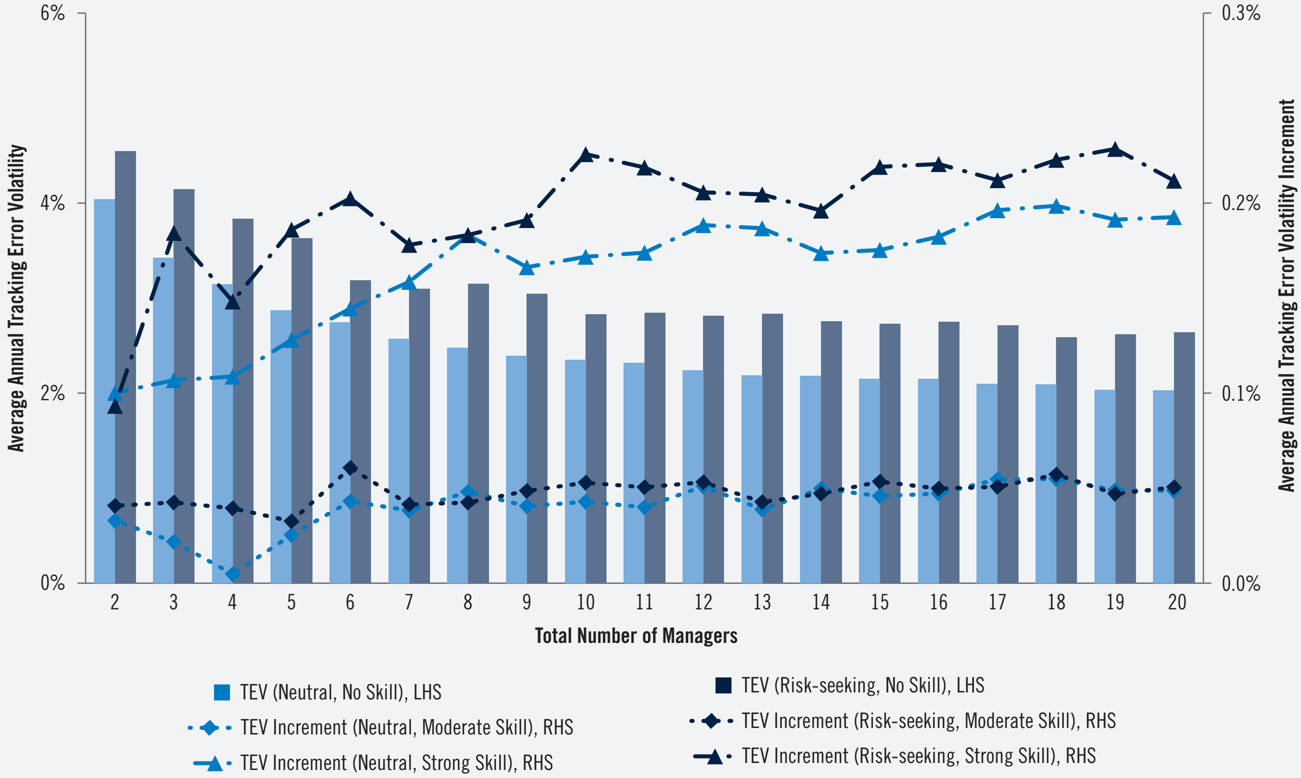
Figure 16: MAP Solutions – Simulated Portfolio Average 3y Annual Alpha and IR
(neutral and risk-seeking preferences; different CIO manager selection skill levels)



Note: The simulation is based on historical manager performance data from January 1997 to December 2017. We show the average result of seven, non-overlapping 3y periods. For a given total number of managers, MAP has multiple solutions. The error bar represents a 95% confidence interval of the average over all solutions. A narrow confidence interval indicates similar performance across MAP solutions. Provided for illustrative purposes only.

Source: eVestment, PGIM IAS.

Figure 17: MAP Solutions – Simulated Portfolio Average 3y TEV
(neutral and risk-seeking preferences)



Note: The simulation is based on historical manager performance data from January 1997 to December 2017. We show the average result of seven, non-overlapping 3y periods. TEV increment is relative to the TEV realized by zero skill, with corresponding preferences. Provided for illustrative purposes only.

Source: eVestment, PGIM IAS.

Since the portfolio average 3y IR curves (versus the total number of managers) are roughly the same between Risk-seeking and Neutral preferences, the optimal number of managers for Risk-seeking CIOs are equal to that for Neutral CIOs: 3-4 managers in the case of no skill, and 8-9 in the case of moderate or strong skill.

Combining Passive and Active Strategies

MAP addresses the active manager allocation problem as it assumes capital is invested in active strategies. The passive vs. active allocation decision depends on what the CIO expects about the active managers' performance in the forward investment horizon, the CIO's portfolio active risk budget and active management cost. Active strategies can add value if the expected alpha and tracking error volatility satisfy the CIO's active return-risk trade-off. A CIO skillful in selecting managers is more likely to realize higher alpha, and therefore would probably prefer active strategies.

Figure 18 summarizes the impact of allocations to passive strategies defined as strategies that track the benchmark closely with low cost.[11] A common concern of active strategies is the uncertain downside, which can be mitigated by passive benchmark-trackers with a constrained upside as a price. If a CIO is worried about the risk of underperforming, passive strategies with a constant cost and negligible tracking error volatility may be suitable.

11 We assume the passive strategies cost 15 bp/y with annual tracking error volatility 0.1%. Other options include quantitative index enhancement strategies with a high breadth and low cost.

Figure 18: Combining Passive and Active Strategies

Manager Selection Skill	Allocation to Passive Strategies	Neutral (Balanced) Preferences				Tilted (Risk-seeking) Preferences			
		Downside Average (bp/y)	Upside Average (bp/y)	Hit Rate	Annual TEV	Downside Average (bp/y)	Upside Average (bp/y)	Hit Rate	Annual TEV
No Skill (4 Managers)	0%	-167	292	50%	3.15%	-182	362	52%	3.84%
	10%	-151	263	50%	2.83%	-164	326	52%	3.45%
	20%	-135	234	49%	2.52%	-147	290	51%	3.07%
	30%	-119	204	49%	2.20%	-129	253	51%	2.69%
	40%	-103	176	48%	1.89%	-112	217	50%	2.30%
	50%	-87	146	47%	1.57%	-94	181	50%	1.92%
Moderate Skill (8 Managers)	0%	-104	270	58%	2.52%	-119	346	58%	3.19%
	10%	-94	243	58%	2.27%	-108	311	58%	2.88%
	20%	-85	216	57%	2.02%	-97	276	58%	2.56%
	30%	-75	189	57%	1.77%	-86	242	57%	2.24%
	40%	-65	162	56%	1.51%	-75	207	57%	1.92%
	50%	-55	135	55%	1.26%	-64	173	56%	1.60%
Strong Skill (8 Managers)	0%	-66	295	77%	2.66%	-75	383	76%	3.34%
	10%	-60	265	77%	2.39%	-68	345	75%	3.00%
	20%	-54	236	76%	2.13%	-61	306	75%	2.67%
	30%	-48	206	76%	1.86%	-55	268	74%	2.34%
	40%	-42	176	75%	1.60%	-48	230	74%	2.00%
	50%	-36	147	74%	1.33%	-41	191	73%	1.67%

Note: We show the result estimated by simulation based on historical manager performance data from January 1997 to December 2017. Hit rate is the percentage of simulation paths scoring positive alpha, net of all cost. Downside/upside average is the average annual net alpha conditioning on the negative/positive side. The average is calculated across all paths of the seven, non-overlapping 3y periods. The active strategy for each manager selection skill level is formed following MAP with the optimal total number of managers. We assume the total investment capital $1b and the cost of passive benchmark-tracking strategies is 15 bp/y with 0.1% annual tracking error volatility. The passive/active blends which generated downside average closest to -100 bp/y are shaded.

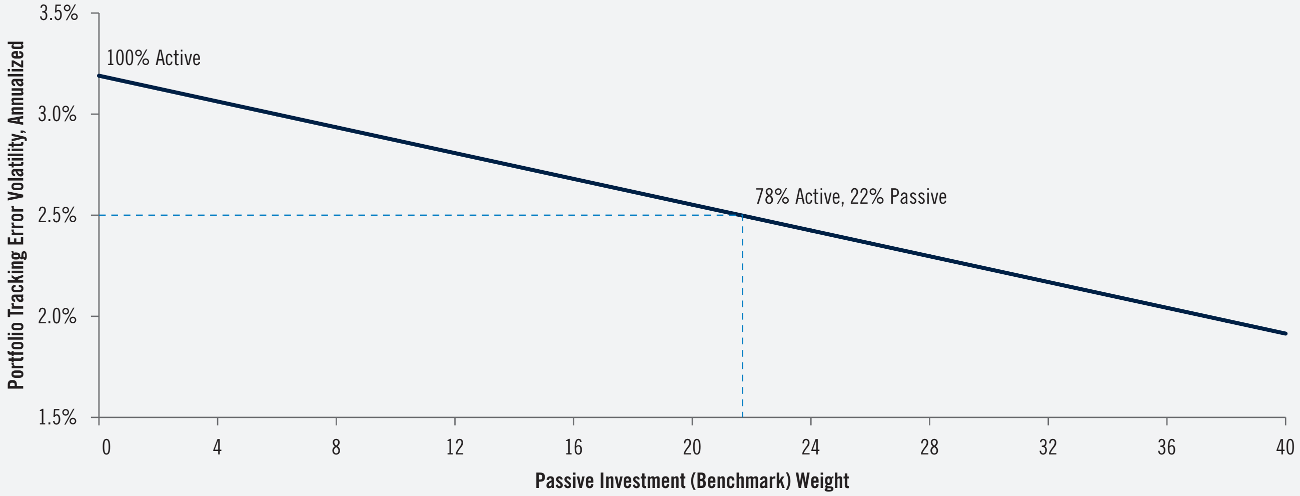
Source: eVestment, PGIM IAS. Provided for illustrative purposes only.

For downside control (*e.g.*, 1%/y downside average as indicated in Figure 18) we find a bulk allocation to passive (40% for Neutral preferences and 50% for Risk-seeking preferences) is reasonable for a CIO with little skill. However, as the CIO's manager selection skill improves, the sacrifice of the upside becomes uneconomic. Especially for a CIO with strong skill, a 100% active portfolio can meet the requirement of 1%/y downside constraint, even though it has relatively high tracking error volatility (2.66% for Neutral preferences and 3.34% for Risk-seeking preferences).[12]

12 Thinking from the bigger asset-liability perspective, we see another incentive for the CIO to assume active risk. In general, the allocation to non-liability asset leads to strategic risk (*i.e.*, risk of not tracking changes in liability values). The active management risk may be helpful for the purpose of diversification. If we assume the CIO has skill uncorrelated to strategic risk, lower overall allocation to equity and higher skill the CIO possesses would encourage more aggressive active risk budgeting.

A CIO should decide their portfolio active risk corresponding to their manager selection skill. With superior selection skill, a reasonably aggressive active risk budget is warranted.[13] Once the target active risk is determined, the passive strategy can be utilized as the "(active) risk free" asset to dilute the overall portfolio tracking error volatility, if it is too high, to the desired level. For example, suppose a CIO with moderate skill and Risk-seeking preferences targets 2.5% portfolio active risk. Given that the simulated annual TEV is about 3.2%, one way to match the active risk budget is to alter the preferences and perhaps scale down the allocation over high active risk managers. However, an allocation to a passive strategy may be a better alternative. As illustrated in Figure 19 including a passive allocation would drive down the portfolio active risk. Roughly, a 20% allocation shrinks the tracking error volatility to the 2.5% target. The active portion of the portfolio is constructed following MAP solutions and accurately represents the CIO's preferences.

Figure 19: Allocation (%) to Passive Strategy and Portfolio TEV



Note: We assume the CIO has Risk-seeking preferences and moderate manager selection skill. MAP solutions for total eight managers are applied to construct the active strategy portion of the overall portfolio.

Source: PGIM IAS. Provided for illustrative purposes only.

13 The optimal active risk budget maximizing the value-added of active management is proportional to the portfolio information ratio (Grinold, 1989). We show that portfolio information ratio increases as the CIO's manager selection skill improves.

Conclusion

We address the construction of multi-manager US large cap equity portfolios. Instead of using traditional mean-variance optimization, we approach portfolio construction from a diversification perspective. We take into consideration a CIO's preferences, any ability to distinguish outperforming managers from peers, and the cost of implementing such a portfolio. We answer the question *What is the optimal number of managers?* by addressing two sub-problems. We first divide the universe of active managers and employ MAP to build a portfolio, for a given total number of managers, which reflects a CIO's preferences and is diversified across manager types. Then for different total number of managers, we simulate portfolio performance. The simulation is based on historical manager performance data over a long period (January 1997- December 2017). The optimal number of managers is determined by expected portfolio IR and alpha net of all cost, both estimated by simulation average. Compared to a CIO with no manager selection skill, we find that a CIO possessing selection skill would benefit from a portfolio with relatively more managers.

We compare the MAP allocations with other manager allocation methodologies, Core-only and Core-satellite (style), and show that the MAP manager allocations lead to a multi-manager portfolio with an efficient active risk-return trade-off. It constrains any unintended concentrations on generic ideas which co-move during periods of market turmoil. Consequently, MAP achieves more consistent portfolio performance across distinct market regimes, helping CIOs wary of the "perception" of diversification.

Our analytical framework is flexible to incorporate other manager characteristics one may find effective to differentiate strategies. It can be potentially applied to construct multi-manager portfolio in other investment mandates, *e.g.,* global equity and fixed income.

Acknowledgements

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APPENDIX

Manager Active Risk Level Inference

A portfolio manager's absolute active risk is determined by many characteristics, including risk budget, expected active return, investment approach and manager style. To isolate the impact of other dimensions, we consider a manager's active risk level in a relative manner: the active risk rank to peers applying similar methodologies and therefore classified in the same group. A manager's relative active risk level, although subject to occasional fluctuation due to either market volatility or the manager's tactical decisions, should be stable over time since it reflects the idiosyncratic component of the manager's total active risk budget. Although not reported in the database we use, a manager's active risk can be estimated with reasonable confidence by CIOs through manager interviews. We infer a manager's active risk level (relative to its peer group) from the excess return time series.

We first estimate the absolute active risk, *i.e.*, tracking error volatility, for each manager in a peer group. The estimation is conducted at the end of each year, with a 3y look back period (36 monthly returns). At each year end we calculate the rank for each manager relative to all others in the same group. Then for each manager, we average the rank to peers across time. A manager will be identified as low active risk level if the average ranking of tracking error volatility is in the bottom 60%; otherwise the manager will be classified as high active risk level. The 60% break point is discretionary. We are cautious to classify a manager as high active risk level, while keeping the classification not too skewed.

Active risk level combined with investment approach and style, partitions the US large-cap equity manager universe as shown in Figure A1.

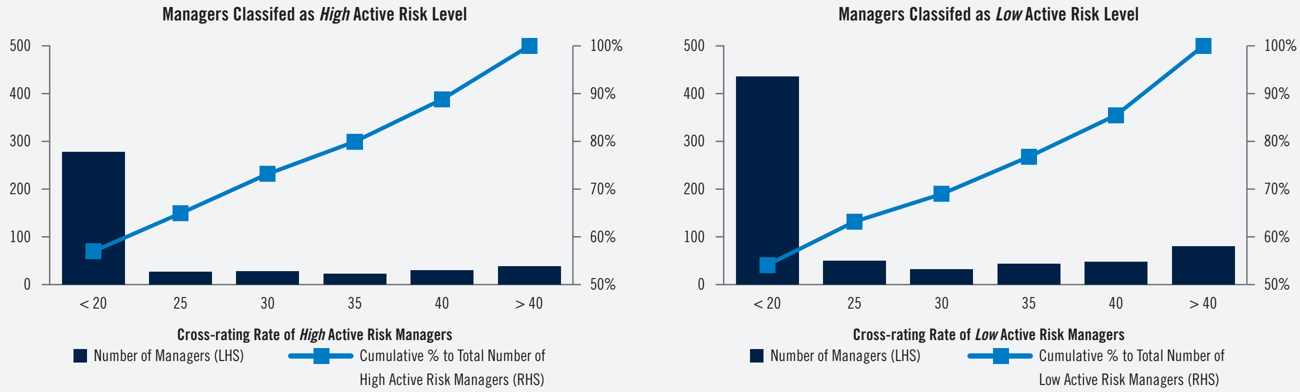
We expect the active risk level categorization to be reasonably stable over time. A manager identified as high active risk level to peers, should not bounce back and force across the 60% threshold over different periods. Figure A2 shows the distribution of manager cross-rating rate, the percentage of periods that a high/low active risk level manager gets an inconsistent assessment in the estimation window (*e.g.* a high active risk manager ranks in the bottom 60% relative to peers in the 2003- 2005 estimation window). More than 65% managers with high/low active risk level have a cross-rating rate of less than 20%. This number increases to 80% for high active risk managers (75% for low active risk managers) if a 30% cross-rating rate is acceptable. This result implies that our inference to a manager's relative active risk level is robust over time, though not perfect. About 10% managers classified as high/low active risk level have cross-rating rate higher than 40%. Managers with high cross-rating rate are often those with a short return time series, making it difficult to infer a manager's relative rank of active risk to peers.

Figure A1: Number of Managers in Each Group

Manager Style	Investment Approach	Active Risk Level	Number of Managers
Core	Fundamental	High	107
		Low	162
	Quantitative	High	60
		Low	76
Growth	Fundamental	High	99
		Low	190
	Quantitative	High	23
		Low	37
Value	Fundamental	High	104
		Low	173
	Quantitative	High	31
		Low	51

Source: eVestment, PGIM IAS. Provided for illustrative purposes only.

Figure A2: Distribution of Manager Cross-rating Rate



Source: PGIM IAS. Provided for illustrative purposes only.

Manager Allocation Programming (MAP)

We introduce the MAP in matrix form.

Suppose the investor considers *Q* dimensions, which are well defined for each manager. $y^* \in R^Q$ represents the target exposure reflecting the CIO's preferences. Given the total number of managers *k*, we solve for *S*, the set of allocations that match, or approximate, the target exposure *y**.

More exactly, we solve an optimization problem:

$$Min_{\theta}\ f(\theta)$$

subject to:

$$\theta \in N^P$$
$$\theta \geq 0$$
$$\theta^T e = k$$

This is an integer programming problem since we require integer solution for θ. The integer constraint is insignificant for large *k*, but has pronounced impact when *k* is within the scope of our discussion ($k = 2,3,\ldots,20$). *P* is the number of groups we categorize the managers. *e* is a vector of size *P*, with all elements equal to one. θ represents the manager allocation over *P* groups. $f: N^P \to R$ is the objective function:

$$f(\theta) = (h(\theta) - y^*)^{\mathrm{T}}\ \Omega\ (h(\theta) - y^*) + \lambda s(\theta)$$

$$h(\theta) = \frac{F\theta}{\theta^T e} = \frac{F\theta}{k}$$

$h: N^P \to R^Q$ calculates the portfolio exposure corresponding to θ. $F \in R^{Q \times P}$ is a loading matrix and each column records the dimension exposure of managers in each group. Therefore $h(\theta) - y^*$ is the deviation from target exposure.

f penalizes deviations, but not necessarily equally. The penalty multiplier is defined by $\Omega \in \mathrm{R}^{Q \times Q}$, which is a semi-positive definite matrix. The concentration of an allocation is also penalized. λ controls the magnitude of this concentration penalty. $s(\theta)$ measures the imbalance of the manager allocation across groups. We use the squared L2 norm of demeaned θ standardized by *P*, *i.e.*

$$s(\theta) = \frac{\hat{\theta}^T \hat{\theta}}{P}$$

$$\hat{\theta} = \theta - \frac{k}{P}\ e$$

To numerically solve this problem, we partition the field of interest and solve the optimization in each small area. Therefore, we capture all allocations best matching the target exposures.

We consider three dimensions: manager style, investment approach and active risk level ($Q = 3$). The three manager characteristics partition the manager universe into twelve groups ($P = 12$). The column of F reflecting manager style exposure takes values of 1/0/-1, representing Growth/Core/Value style. The other two columns reflecting exposure on investment approach and active risk level, taking values of 1/-1, representing quantitative/fundamental and high/low active risk level, respectively. The target exposures of the two different CIO preferences, as discussed in the article, are

$$y^*_{Neutral} = (0, 0, 0), and$$

$$y^*_{Risk-seeking} = (0.2, -0.2, 0.5)$$

We set Ω as a diagonal matrix, ignoring the penalty to interactions of deviations. The relative magnitude of diagonal elements determines the importance of the corresponding dimension. Larger element implies a higher priority in the optimization. We set Ω as below, with decreasing priority for manager style, active risk level and investment approach:

$$\Omega = \begin{pmatrix} 12_{[manager\ style]} & 0 & 0 \\ 0 & 2_{[investment\ approach]} & 0 \\ 0 & 0 & 5_{[active\ risk\ level]} \end{pmatrix}$$

We set small $\lambda = 1/k$ to differentiate non-concentrated solutions while keeping the focus on minimizing portfolio exposure deviations.

MAP is flexible and can accommodate customized settings. Both the Neutral and Risk-seeking preferences we discussed in this article set relatively loose constraints and therefore result in multiple feasible solutions. The CIO can tune the parameter, and add other constrains, *e.g.*, excluding certain manager groups or requiring minimal allocations to core managers so as to eliminate barbell-type portfolios.

Modeling Manager Selection Skill

We assume the CIO would have a higher/lower probability to select the high/low ranking managers as skill improves. More exactly, the probability density function is

$$f(x) = \frac{x^v}{\int_0^1 x^v dx} \quad , \quad 0 < x < 1$$

where x is the relative ranking of the manager compared to peers. The ranking is calculated by the cumulated excess return to the benchmark in the forward investment horizon (3y). v measures the manager selection skill level and plays as the "twist factor" determining the shape of the probability density function of manager performance relative rank. Figure A3 shows the probability of a selected manager, by quartile, with the three skill levels discussed in the article.

Figure A3: Probability of a Selected Manager in Each Ranking Quartile

Manager Selection Skill	1st Quartile (Worst)	2nd Quartile	3rd Quartile	4th Quartile (Best)
No Skill (v = 0)	25%	25%	25%	25%
Moderate Skill (v = 0.2)	19%	25%	27%	29%
Strong Skill (v = 0.7)	10%	21%	31%	39%

Note: Sum of % from all quartiles may not equal to 100% due to rounding.
Source: PGIM IAS. Provided for illustrative purposes only.

Simulation Setting

We consider the 21y history from January 1997 to December 2017 and partition it into seven, non-overlapping 3y investment periods. In each period, we consider different combinations of total number of managers (from 2 to 20), CIO's preferences (Neutral and Risk-seeking), manager selection skills (zero, moderate and strong skill) and different allocation methodologies (Core-only, Core-satellite (style) and MAP). For each combination, we simulate 4,000 scenarios to randomize the manager selection results.

At the beginning of each scenario, the CIO first chooses a manager allocation. If there are multiple feasible allocations, one is randomly picked. The CIO faces the same candidate portfolio managers across different scenarios and applies selection skill. The candidate manager pool includes all managers who are active at the beginning of each 3y period. Managers are selected with the probability determined by the CIO's selection skill. If a selected manager discontinues business (*i.e.,* stops reporting performance) during a period, the invested capital is redeemed at current value, and the CIO will repeat the manager selection process at identical skill level to hire another active manager in the same peer group. We assume the replacement happens in the following month, and a one-time penalty, 50bp of the redeemed value, is charged to reflect the manager transition cost.

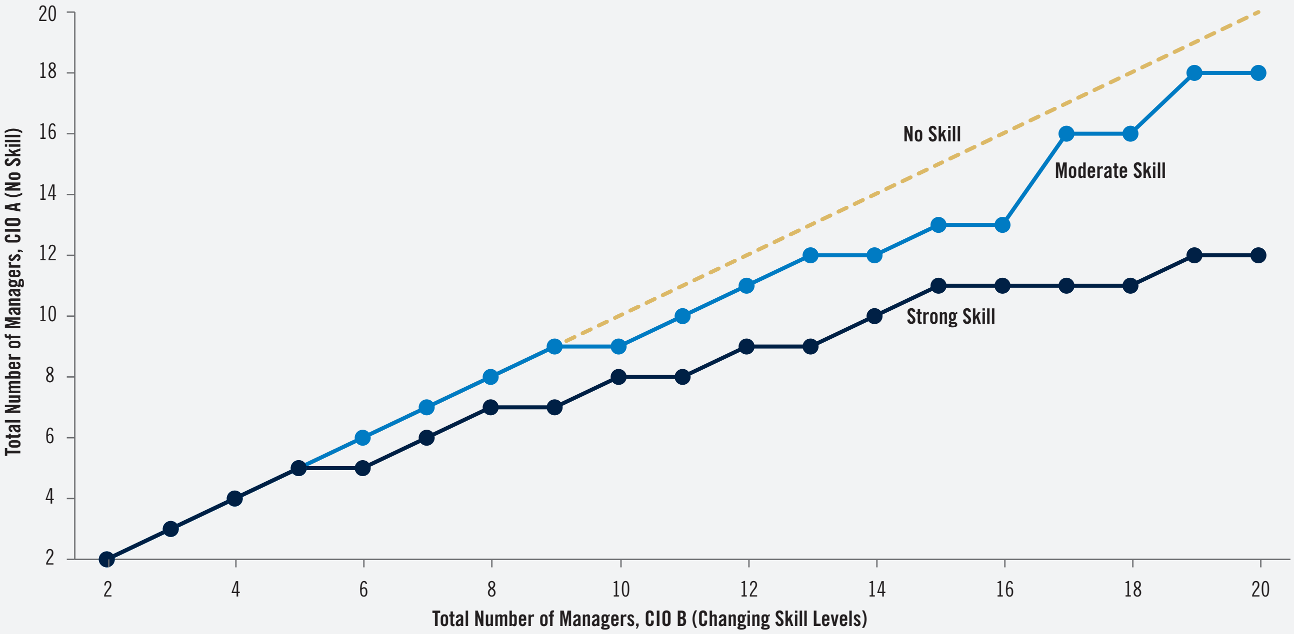
We record investment performance, both gross and net of cost, for each scenario. We calculate the average over 4,000 scenarios to compare the influence of different combinations in a specific period. We further average across all 3y periods to focus on long-term impact.

Portfolio Tracking Error Volatility (TEV) with Manager Selection Skills

We compare the impact of a CIO's manager selection skill on portfolio tracking error volatility, with different total number of managers.

Figure A4 shows a number-number plot, illustrating how many managers can diversify the portfolio TEV to the same level achieved by a no-skill CIO, assuming the MAP allocation methodology is applied. The line of equivalent TEV gets bent down, from the 45-degree diagonal line (representing the no-skill-to-no-skill case), as the manager selection skill improves. The portfolio TEV of a no-skill CIO declines faster than that of a skillful CIO.

Figure A4: Number of Managers to Match Portfolio TEV



Note: We consider the portfolio TEV for Neutral preferences achieved by MAP allocation solutions with different manager selection skills. The TEV is estimated by simulation based on historical manager performance data from January 1997 to December 2017. We show the average annualized TEV of seven, non-overlapping 3-year periods. The vertical axis shows the total number of managers CIO A (no skill) would hire to generate a portfolio with TEV matched by CIO B (different skill levels) with total number of managers represented by the horizontal axis.
Source: eVestment, PGIM IAS. Provided for illustrative purposes only.

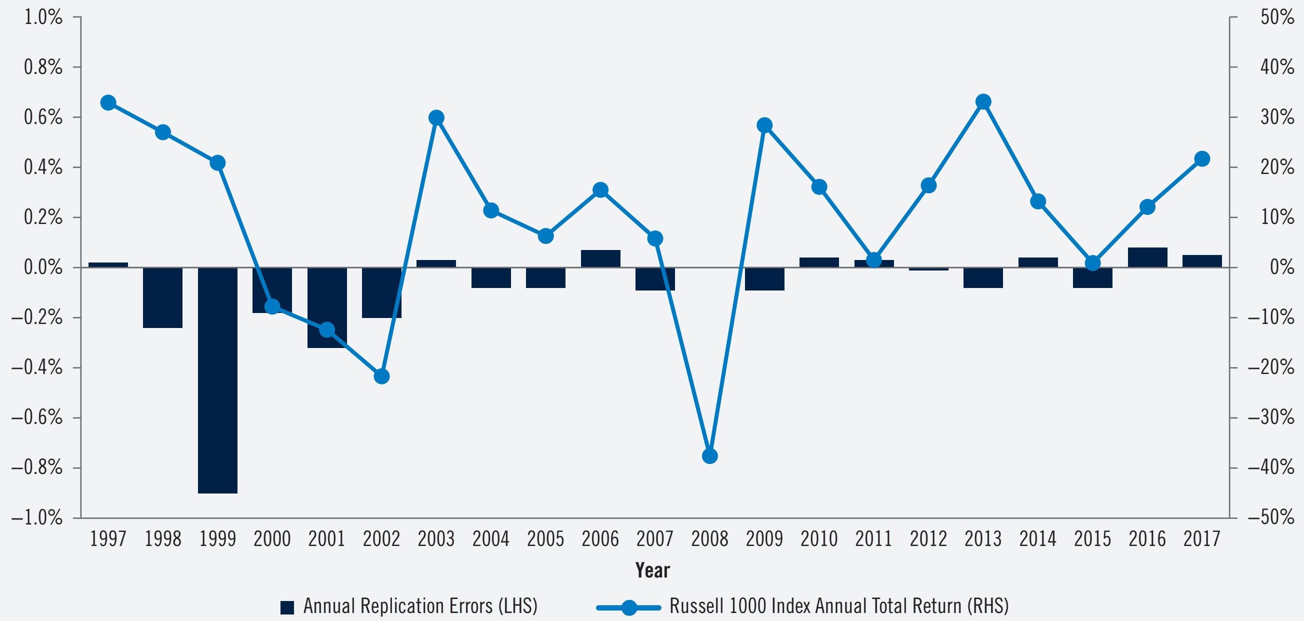
Style-tilted Benchmark by Combining Russell 1000 Indices

We use the Russell 1000 Index as the benchmark for US large-cap Core equity managers and for CIOs who would like to have core style exposures. Russell 1000 Growth/Value Index is used to benchmark US large-cap Growth/Value equity managers and for CIOs fully (*i.e.,* 100%) tilted to growth/value in style. For CIOs who are partially tilted in style, we construct a customized benchmark to reflect the same style exposure by combining the Russell 1000 indices.

According to the index construction, stocks are always fully represented by the combination of their growth and value weights.[14] Therefore a 50% investment in Russell 1000 Value Index plus 50% investment in Russell 1000 Growth Index would replicate the Russell 1000 Index. Figure A5 illustrates the annual accumulated replication accuracy. Errors are attributed to rebalancing schedules and index inclusion/exclusion events.

For a CIO with a 20% tilt to growth preference, for example, we construct a customized benchmark as 60% in Russell 1000 Growth Index and 40% in Russell 1000 Value Index for the purpose to accurately measure the portfolio's active performance. The customized benchmark is investable as the components are well tracked by low-cost, transparent passive products.

Figure A5: Annual Replication Errors
(50% Russell 1000 Value plus 50% Russell 1000 Growth, against Russell 1000)

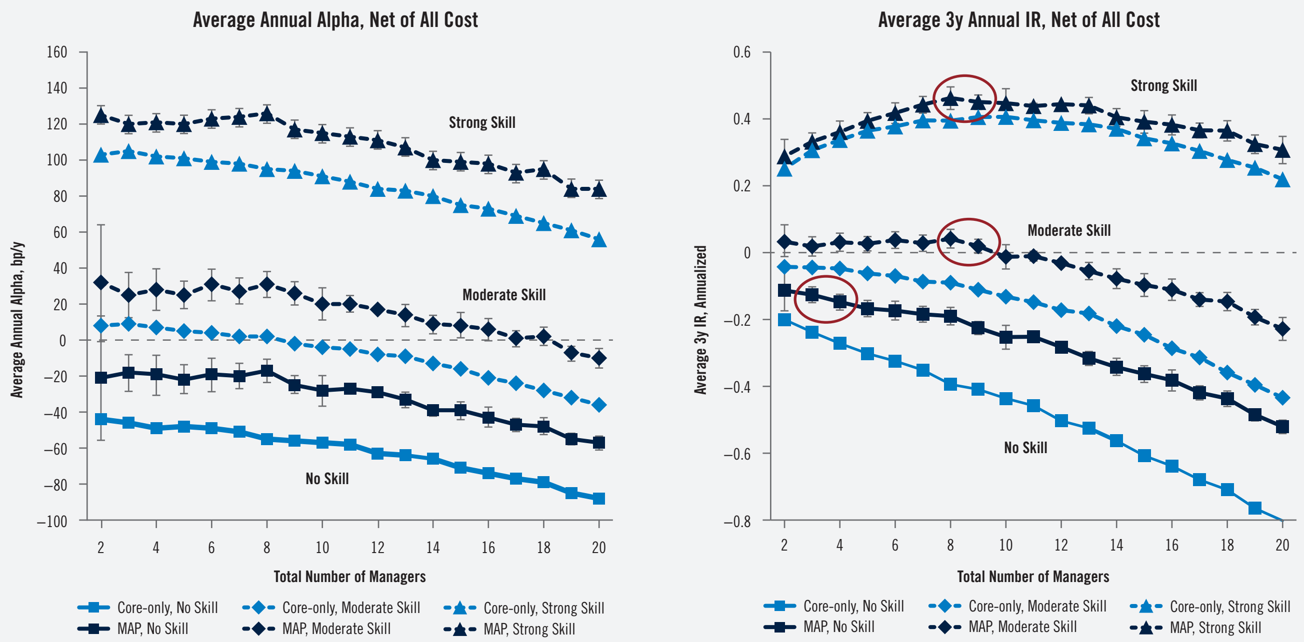


Source: Datastream, PGIM IAS. Provided for illustrative purposes only.

14 FTSE Russell, Russell U.S. Equity Indexes Construction and Methodology, version 4.0, August 2019.

Simulation Result, Excluding January 2000 – December 2002 Period

Figure A6: Simulated Average Portfolio 3y Alpha and IR
(neutral preferences; different CIO manager selection skill levels; excluding January 2000 – December 2002)



Note: The simulation is based on historical manager performance data from January 1997 to December 2017, excluding January 2000 – December 2002. We show the average result of seven, non-overlapping 3y periods. For a given total number of managers, the Core-only method produces one manager allocation solution while MAP has multiple solutions. The error bars of MAP solutions represent 95% confidence intervals of the average over all solutions. A narrow confidence interval indicates similar performance across MAP solutions. The information ratios, achieved by the optimal number of managers following the MAP method with each of the three levels of manager selection skill, are marked by red circles. Provided for illustrative purposes only.
Source: eVestment, PGIM IAS.

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