PGIM | GIC

BUILDING A BETTER PORTFOLIO

Balancing Performance and Liquidity

APRIL 2020

PGIM INSTITUTIONAL ADVISORY & SOLUTIONS

AUTHORS

Junying Shen
junying.shen@pgim.com
+1 973 367 8198

Ding Li
liding@gic.com.sg
+65 6889 8233

Grace (Tiantian) Qiu, PhD
graceqiu@gic.com.sg
+65 6889 8223

Vishv Jeet, PhD
vishv.jeet@pgim.com
+1 973 367 6859

Michelle (Yu) Teng, PhD, CFA
michelle.teng@pgim.com
+1 973 367 6947

Ki Cheong Wong, PhD
wongkicheong@gic.com.sg
+65 6889 5315

This research is a collaboration between GIC EIS and PGIM IAS.

ABOUT PGIM IAS

The PGIM Institutional Advisory & Solutions group advises institutional clients on a variety of asset allocation and portfolio construction topics, and delivers bespoke research based on an institution's specific objectives. For inquiries and to learn more about PGIM's investment advisory capabilities, email IAS@pgim.com or visit pgim.com/IAS.

ABOUT GIC EIS

GIC is a leading global investment firm established in 1981 to manage Singapore's foreign reserves. As a disciplined long-term value investor, GIC is uniquely positioned for investments across a wide range of asset classes, including equities, fixed income, private equity, real estate and infrastructure. GIC's Economics & Investment Strategy department (EIS) conducts bespoke economic and long-term thematic research, and guides GIC's top-down portfolio design and management by constructing long-term portfolio investment policy, defining strategic asset allocation, undertaking medium-term strategy, as well as innovating alternative investment models. For more information on GIC, please visit www.gic.com.sg.

BUILDING A BETTER PORTFOLIO

Balancing Performance and Liquidity

Investors have been increasing their allocations to private assets seeking higher returns and better portfolio diversification. However, as this allocation increases, the liquidity characteristics of their portfolios change. To address this issue we create a framework that links bottom-up private asset investing with top-down asset allocation. Private asset cash flows are consistently modeled together with public asset returns and risk that, in turn, drive portfolio construction. This helps investors analyze how allocations to illiquid private assets, in combination with their commitment strategy, may affect their portfolio's ability to respond to various liquidity demands. By measuring the potential tradeoff between asset allocations, total portfolio performance and the frequency of certain liquidity events with different severities, this framework can help investors quantify the interaction between their portfolio structure and performance, and formalize their decision making around portfolio liquidity choices.

The search for higher returns and better diversification has led many institutional investors to allocate more capital to illiquid private assets. This has come at the cost of decreasing portfolio liquidity, as private assets are not easily sold in a short period of time and may be unable to meet immediate portfolio liquidity demands. At the same time, private asset investors may encounter additional and often hard to predict liquidity demands when GPs make capital calls stemming from prior commitments. Investors need to have a strong understanding of how the liquidity characteristics of private assets impact their portfolios.

For asset allocators, liquidity risk is one of the most critical, but least quantified, risk dimensions in portfolio construction. Traditional portfolio construction techniques including mean-variance optimization or risk parity focus heavily on return variability and drawdowns, but often treat liquidity risk as a secondary consideration. Unlike fluctuations in returns, which tend to have a transitory impact, liquidity can be a matter of survival. Balance-sheet sustainability and funding stability are of critical importance to all investors. Institutional investors with required periodic obligations (*e.g.*, public and private pension plans) need to ensure that their asset allocation does not unduly risk meeting these obligations. Even investors without explicit obligations (*e.g.*, some sovereign wealth funds) may have critical liquidity needs such as rebalancing the portfolio to manage risk or having enough dry powder to provide support during periods of market dislocation.

We have enhanced and expanded PGIM's asset allocation framework (OASIS™ – **O**ptimal **As**set Allocation with **I**lliquid Asset**s**) that can help investors analyze how allocations to illiquid private assets, in combination with their commitment strategy, may affect their portfolio's ability to respond to liquidity demands. By measuring the potential tradeoff between asset allocations, portfolio performance and the frequency of certain liquidity events with different severities, this framework allows investors to quantify the interaction between their portfolio structure and performance, and formalize their decision making around portfolio liquidity choices.

Much has been published separately on the two topics covered by this paper: private asset cash flow modeling and portfolio construction with illiquid private assets. For many institutional investors, the former is well understood by their private deal teams and is often modeled on a deal-by-deal basis or at the aggregated strategy/vintage level, while the latter is conducted by the team responsible for top-down asset allocation. The limitation of such an arrangement is that portfolio asset allocation decisions often do not consider the bottom-up cash flow information and, likewise, the deal teams usually do not formulate their commitment strategies in a total portfolio context. Consequently, the portfolio liquidity implications of the combined decisions of the two groups are rarely explicitly modeled.

The unified framework introduced in this paper links private asset cash flow modeling with asset allocation analysis. The private asset cash flows are consistently modeled together with expected public asset returns and risk that drive the portfolio construction process. By doing so, liquidity measurement and cash flow management can be formally integrated into a multi-asset, multi-period portfolio construction process. In addition, appropriate commitment strategies can be designed while simultaneously considering the portfolio's desired liquidity characteristics.

Last but not least, the framework is flexible and customizable, allowing investors to incorporate their own assumptions regarding:

- Public asset performance and risk (beta and alpha);
- Private asset performance and risk (relative to public assets) and fund-selection skills;
- Private asset commitment strategy;
- Total portfolio cash flow needs and liquidity supply waterfall; and
- Penalty for various liquidity events.

I. LITERATURE REVIEW

Research related to private asset cash flows generally falls into two categories: cash flow prediction models and private asset commitment strategies. As institutional allocations to private assets began to grow significantly, the need for **cash flow prediction models** arose. In response, Takahashi and Alexander (2002) proposed their cash flow model (*i.e.*, the Yale model) which is among the earliest work formalizing a deterministic predictive model that captures the stylized pattern of LP capital contributions, distributions and NAVs. de Malherbe (2004, 2005) then developed a continuous-time stochastic version of the Yale model with the specification of the dynamics of unobservable fund values. Buchner, *et al.* (2010) proposed another stochastic model to better match various typical drawdown and distribution patterns. Meads, *et al.* (2016) focused on building a model for predicting capital calls based on the idea of internal age of contributions (Mathonet and Meyer, 2008). O'Shea and Jeet (2018a) examined the effect of market crises on private capital cash flows. Nonetheless, most private asset cash flow models do not offer an intuitive linkage to market dynamics and uncertainty.

Lerner, *et al.* (2007) investigated why the returns that institutional investors realize from private equity differ dramatically across institutions. They suggest that an important factor explaining this performance variation (beyond the usual factors such as experience, sophistication, access, size, geography and industry focus) is the investment objective. A good **private asset commitment strategy** is crucial to balance several investment objectives including performance, risk and liquidity. To build up as well as maintain a desired allocation to the asset class, Cardie, *et al.*, (2000) provided a rule of thumb: to commit half of the capital allocated to private assets each year. This is a deterministic rule that disregards any currently available information. Alternatively, Zwart, *et al.* (2012) and Oberli (2015) suggested simple commitment strategies that are based on current information such as the amount of uncalled capital, cash, NAV, and recent distributions. Their idea is to commit a fraction of the overall capital allocated to private assets that is not "in the ground." They compute the fraction as the ratio of NAV to NAV plus cash. If there is no cash, then the fraction is one and the entire distribution and some of the uncalled capital (older than 6 years) is committed again. Nevins, *et al.* (2004) provided a commitment model that uses four parameters: 1) rate of capital calls; 2) rate of distribution; 3) rate of return on public assets; and 4) rate of return on private assets.

Regarding **portfolio construction with illiquid private assets**, which is the key topic covered by this paper, Lo, *et al.* (2003) introduced liquidity as an explicit constraint or additional dimension in the mean-variance optimization process. Kinlaw, *et al.* (2013) proposed that investors treat liquidity as a shadow allocation within a portfolio, mapping units of liquidity onto units of expected return and risk. Using this theoretical framework Van Luu, *et al.* (2014) performed a case study to estimate how much additional return would be required to take on liquidity risk given the level of expected liquidity demands.

These latter two papers are perhaps the most closely related to ours, in terms of motivations and objectives. These papers focus on quantifying the shadow price of liquidity as an explicit and comparable tradeoff with portfolio return, while ours is a framework that explicitly captures liquidity events with different severities, and treats portfolio liquidity as a distinct element to be traded off with portfolio performance. In addition, investors can adjust the model setup based on their own asset and liability profiles, potential liquidity demands, and the liquidity supply waterfall based on their investment processes. Overall, the asset allocation framework can help answer liquidity questions from various investors.

II. MOTIVATION

Being prepared to respond to liquidity demands has implications for a portfolio's structure and, hence, expected portfolio performance. At one extreme, holding just cash may give an investor full flexibility to meet unexpected liquidity demands, but will likely hurt performance. At the other extreme, holding illiquid (but expected high performing) assets might give the investor little room to meet liquidity demands and may ultimately hurt performance if these illiquid assets must be sold prematurely, often at a discount, to meet cash needs. The challenge for the investor is how to structure the portfolio in a way that maximizes expected portfolio performance while keeping liquidity events under control, in terms of both frequency and severity. In short, private asset investors are faced with several important questions:

1) How to formulate a **private asset commitment strategy** to manage private asset exposure and the uncertainty in timing and magnitude of their cash flows over time?

2) What should be the **desired allocations** (public *vs.* private, public passive *vs.* public active) given the investor's liquidity risk tolerance?

3) How would various **market scenarios** impact the portfolio's liquidity and performance?

We develop a cash flow-driven asset allocation framework to help investors answer these questions and conduct portfolio liquidity analysis. The framework explicitly incorporates the unique characteristics of private assets such as the delay and uncertainty of capital calls, lumpy and high transaction costs and high idiosyncratic risk. For private assets, the framework distinguishes LP allocation value (what the investor experiences) from LP investment value (what the GP reports) by taking into account the performance of any undrawn capital invested in a "default public investment" until called by the GP. The horizon value of an LP allocation (or an LP investment) is affected by the timing and magnitude of the capital calls which are at the GP's discretion. In addition, the framework allows investors to express their views on expected private asset performance relative to public markets as well as their fund-selection skill which can be an important driver of private asset performance.

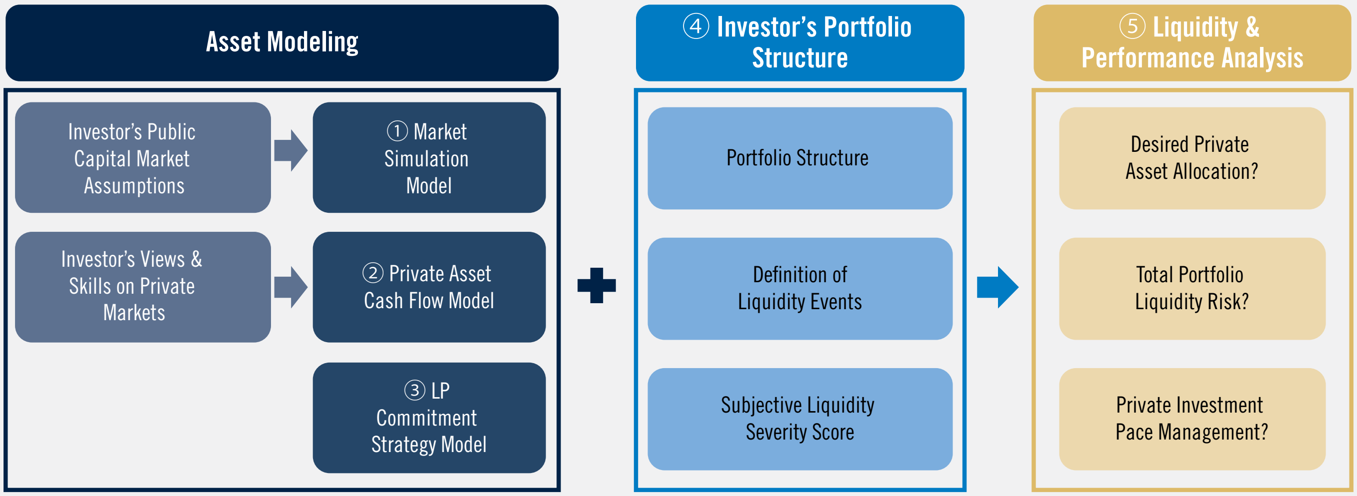
To simplify the analysis, we consider an institutional portfolio with no required scheduled payment obligations, *e.g.*, a sovereign wealth fund.[1] Nevertheless, such a portfolio may still have significant liquidity requirements. One liquidity concern is cash demands from the sovereign sponsor, such as contingent claims in a crisis and strategic investment initiatives. Unexpected cash inflows or outflows could also stem from the objective of a sovereign wealth fund to insulate the budget and economy from commodity price volatility. A fund may also need liquidity to respond to unforeseen market movements and GP capital calls.

III. ASSET ALLOCATION FRAMEWORK

Our asset allocation framework contains five major components (labeled ① to ⑤ in Figure 1). The ① component generates public returns based on an investor's capital market assumptions. The ② and ③ components produce private asset cash flows based on the investor's views on private asset performance, fund-selection skill, and the commitment strategy. The interaction of the returns and private cash flows with the investor's portfolio structure (the ④ component) determines the portfolio's liquidity and expected performance (the ⑤ component). We briefly describe each component and their interactions, leaving the details to the Appendix.

1 For brevity, we focus on liquidity analysis for institutional portfolios with no required scheduled payment obligations. However, the framework can be used by various types of investors with a wide range of liability profiles. An application of the framework for a corporate defined benefit plan (representing institutional investors facing explicit periodic payment obligations) can be found in Teng and Shen (2019).

Figure 1: Flowchart of the OASIS Asset Allocation Framework



Source: GIC EIS & PGIM IAS. Provided for illustrative purposes only.

Market Simulation and Private Asset Cash Flows

We simulate the risk and returns of a multi-asset portfolio, including private and public markets. The process is flexible and can incorporate an investor's own capital market assumptions.

We adopt the Takahashi and Alexander (TA) model and calibrate it to capture some empirical relationships between cash flows and public market performance. A cash flow model that is consistent and responsive to the underlying capital market environment allows investors to perform stress tests and tailor their liquidity analysis to forward-looking scenarios. Our framework also gives investors the flexibility to specify fund characteristics, incorporate their views on both public and private market performance and fund-selection skill.

LP Commitment Strategies

We introduce two types of LP commitment strategies. We discuss each strategy's objective and define how the commitment amount is determined each quarter. We also discuss each strategy's pros and cons.

Cash Flow Matching (CFM)

The CFM commitment strategy aims to build a private asset portfolio whose periodic net cash flows are close to zero. In other words, all distributions received in the previous quarter should fund all capital calls in the next quarter. Such a strategy can help insulate the rest of the portfolio from the private asset investment activity.

The commitment amount at the beginning of each quarter is determined so that the projected net cash flow (distributions minus capital calls) two quarters ahead (based on reported NAV at the end of last quarter) is zero. Specifically, we

- Project two quarters ahead as we assume the first capital call occurs two quarters after the commitment; and
- Use "*no view*" assumptions (*i.e.*, the average value of calibrated vintage-level parameters) for the TA model parameters in cash flow projections.

The CFM commitment strategy has a few limitations: 1) The strategy can lead to a volatile commitment pattern over time and may skip commitments over multiple periods – which may be undesirable for maintaining vintage diversification; 2) The strategy does not have control over how NAV will grow as a percentage of the overall portfolio; and 3) If one is starting a private capital investment program with no prior commitments and NAV, then cash flow matching is not possible until distributions start to arrive.

Target NAV

The Target NAV strategy tries to achieve and maintain a target NAV% of the overall portfolio. It is useful to think of a private portfolio having three distinct pools of capital: 1) The capital that is **"in the ground"** also known as the **NAV**; 2) The **"committed, but uncalled" capital**, which is the capital that is committed but has not yet been called; and 3) The **"uncommitted" capital**, which is the capital initially allocated to private assets and distributions received from prior commitments that has not yet been committed.

The commitment amount at the beginning of each quarter is determined by multiplying a fixed portion (f) by the total amount of uncommitted capital at the end of the prior period. The pool of uncommitted capital is continuously replenished by distributions from prior commitments. The idea of keeping track of uncommitted capital is appealing because it makes the private allocation a *self-contained* portfolio. A target NAV can be achieved by selecting the appropriate f value. Ideally, while f should be set to a value that is less than one to pace out commitments for vintage diversification, f can be set to be greater than one to ramp up NAV.[2]

A drawback of this strategy is that it does not necessarily balance cash flows (especially for f values greater than one), and therefore may require continuous interactions with other parts of the portfolio (*e.g.*, active public asset strategies) – which must either be bought or sold to absorb or free up capital for private market-related cash flows.[3]

Portfolio Structure and Liquidity Events

Portfolio Structure

We assume that investors sort assets in their portfolios by their "transactability" (*i.e.*, ease and cost of selling them to meet a liquidity need). We classify portfolio assets into three types: two liquid types and one illiquid (Figure 2). The two liquid asset types include *liquid passive* assets representing investments in equity and fixed income assets not expected to earn an alpha (*e.g.*, an ETF on a broad-based index fund) and *liquid active* assets that are actively managed to earn an alpha over passive indices (*e.g.*, an actively managed fund or a liquid hedge fund strategy). The *illiquid* private asset type represents all investments in private assets.[4]

Figure 2: Portfolio Structure; Liquidity Demands and Sources; and Waterfall for Sourcing Liquidity

Liquidity Sources for…					… these Liquidity Demands			
Asset Type	Liquidity Level	Liquidity Level Description	Asset		GP Capital Calls	Rebalancing	Dry Powder Creation	Dry Powder Reversal
(1) Liquid Passive	1A	**Committed, but Uncalled** Reserve for Capital Calls **Uncommitted** Reserve for Capital Calls	Stock ETF	Bond ETF	✓	-	-	-
	1B	Available for Liquidity	Stock ETF	Bond ETF	✓	✓	✓	✓
(2) Liquid Active	2	Available for Capital Calls if Level 1 is Exhausted	Stock ETF + αS	Bond ETF + αB	✓	-	-	-
(3) Illiquid	3	Unavailable for Liquidity	LP Investments (NAV only)		-	-	-	-

Source: GIC EIS & PGIM IAS. Provided for illustrative purposes only.

2 Choosing an f that is greater than one means that the strategy is committing more capital than it has. Such a strategy is essentially committing the uncalled capital again to speed up NAV growth and may break the assumed self-contained nature of the private portfolio.

3 One major difference between the CFM and Target NAV commitment strategies is that CFM requires a model for predicting private asset cash flows while Target NAV does not. An alternative strategy may use information about upcoming cash flow events (*e.g.*, potential acquisition or sales) as such information is often informally made available to LPs but is difficult to simulate.

4 In the case study, we assume the private asset is entirely non-transactable (*i.e.*, 100% transaction costs). However, OASIS does allow investors to sell their illiquid private assets and specify their own liquidation rules: 1) The transaction cost of selling private assets; 2) Minimum transaction size to reflect the "lumpiness" of private asset trading; and 3) A lockup period before LP investments (NAV) can be sold or redeemed.

Liquidity Demands, Sources and Events

We recognize four categories of portfolio liquidity demands:

1. **GP Capital Calls:** An obligation that an LP must fulfill based on total initial committed capital amounts, but the timing and amount of each capital call are not under the LP's control;
2. **Rebalancing:** Shift portfolio allocation between public stocks and public bonds at quarter ends to maintain policy or target weights;
3. **Dry Powder Creation:** A tactical move into higher beta assets (*i.e.*, stocks) during market downturns (*i.e.*, at the end of each month if the public equity market experiences a large drawdown) to provide market support or to take advantage of the market dislocation; and
4. **Dry Powder Reversal:** When a market recovery occurs (*i.e.*, when a drawdown is less than -5% following a recovery in the equity market) there is a need to adjust public stocks and bonds back to their initial relative target weights.

An investor must specify a portfolio structure that defines which asset categories will serve as liquidity sources for various liquidity demands and which categories, if any, are considered unavailable (*e.g.*, illiquid private assets). An investor must also specify a "waterfall" rule for sourcing liquidity: First sell assets from the part (or, "liquidity level") of the portfolio that would be least disruptive and expensive. If more assets must be sold, then source liquidity from increasingly disruptive and expensive liquidity levels.

Figure 2 shows that liquid passive assets are sub-divided into two levels with 1A representing a liquidity reserve for GP capital calls and 1B representing the main liquidity source for all other liquidity demands. Since LPs strive to meet GP capital calls, even if it is costly to do so, we assume that calls may draw liquidity from all liquid assets following the waterfall: First, reserve for capital calls (1A), including both "committed, but uncalled" and uncommitted capital, followed by liquid passive assets (1B) and finally liquid active alpha assets (2). The other three types of liquidity demands can source liquidity only from liquid passive assets (1B).

A **liquidity event** occurs whenever an investor must move down the waterfall to find liquidity. For example, a liquidity event would occur if the investor, having exhausted the "liquid passive reserve" portion of the portfolio, must sell liquid active alpha assets. Another liquidity event would occur if the investor also exhausts the liquid active alpha assets and is still unable to fulfill the liquidity demand (if illiquid assets cannot be sold). Consequently, a large liquidity demand could produce a cascade of liquidity events.

Severity of Liquidity Events

Some liquidity events are likely to be of more concern to an investor than others. For example, liquidating liquid active alpha strategies may be more painful due to transaction and opportunity costs than drawing down the liquidity reserve of passive beta assets. Therefore, the framework allows an investor to specify a subjective severity value to each type of liquidity event. This, in turn, allows the investor to ascertain the liquidity severity score of their overall portfolio.

Figure 3 shows an example of the severity value for each type of liquidity event, with a higher value representing a more severe event. Each simulation run has a severity score that is the sum of severity values of all individual liquidity events that occur over the 10y investment horizon. For example, if one simulation run encounters twelve Rebalancing Liquidity Shortage (1B_RB) events and one level 1A Capital Call Liquidity Shortage (1A_CC) event, the severity score for this simulation run is 14 (= (1 × 12) + (2 × 1)). The portfolio liquidity severity score is the average of the 5,000 severity scores across all simulation runs. The portfolio liquidity severity score allows the investor to quantify how their portfolio's liquidity might change with changes to the portfolio structure and commitment strategy.

Figure 3: Liquidity Event Severity Values – An Example

Liquidity Events		Severity Value
Rebalancing Liquidity Shortage	1B_RB	1
Dry Powder Creation Liquidity Shortage	1B_DP	1
Dry Powder Reversal Liquidity Shortage	1B_DP_RB	1
Capital Call Liquidity Shortage (3 types)	1A_CC	2
	1B_CC	3
	2_CC	4

Note: Yellow field indicates an investor input.
Source: GIC EIS & PGIM IAS. Provided for illustrative purposes only.

IV. BRINGING IT ALL TOGETHER

A case study illustrates how a commitment strategy and portfolio structure interact to determine a portfolio's expected performance and liquidity. It also allows us to address the three key questions associated with private asset investing highlighted earlier (see Section II).

Baseline Portfolio Assumptions

We assume the baseline portfolio has an initial AUM of $1b and asset allocation as shown in Figure 4. The (relative) weight for public bonds is 45% within the public portfolio (*i.e.*, 38% public bond allocation (= 1% + 2% + 35%) divided by the 85% total liquid public allocation).[5]

In this example, the illiquid private asset is assumed to be US LP buyout funds.[6] The initial allocation to private assets (NAV) is relatively low (15%). The allocation to liquid passive assets is low (collectively, only 5% in liquid passive stocks and bonds) which will likely produce a higher probability of using up all level 1A assets and consequently produce more Level 1B Rebalancing Liquidity Shortage (1B_RB) liquidity events. Ultimately, the allocation to each level of assets in the portfolio structure is the investor's decision depending on their performance and liquidity objectives.

Figure 4: Baseline Portfolio Structure: Initial Asset Allocation

Asset Type	Liquidity Level	Liquidity Level Description	Stock	Bond
(1) Liquid Passive	1A	**Committed, but Uncalled** Reserve for Capital Calls	1%	1%
		Uncommitted Reserve for Capital Calls	0%	0%
	1B	Available for Liquidity	1%	2%
(2) Liquid Active	2	Available for Capital Calls if Level 1 is exhausted	45%	35%
(3) Illiquid	3	Unavailable for Liquidity (LP Investment NAV)	15%	

Note: Yellow field indicates an investor input.
Source: GIC EIS & PGIM IAS. Provided for illustrative purposes only.

Formulating a Commitment Strategy to Build and Maintain a Desired Allocation

We consider two types of strategies – Target NAV and CFM – and show their cash flow patterns and liquidity consequences. This evaluation helps tackle our first question associated with investing in private assets:

1) How to formulate a **private asset commitment strategy** to manage private asset exposure and the uncertainty in timing and magnitude of cash flows over time?

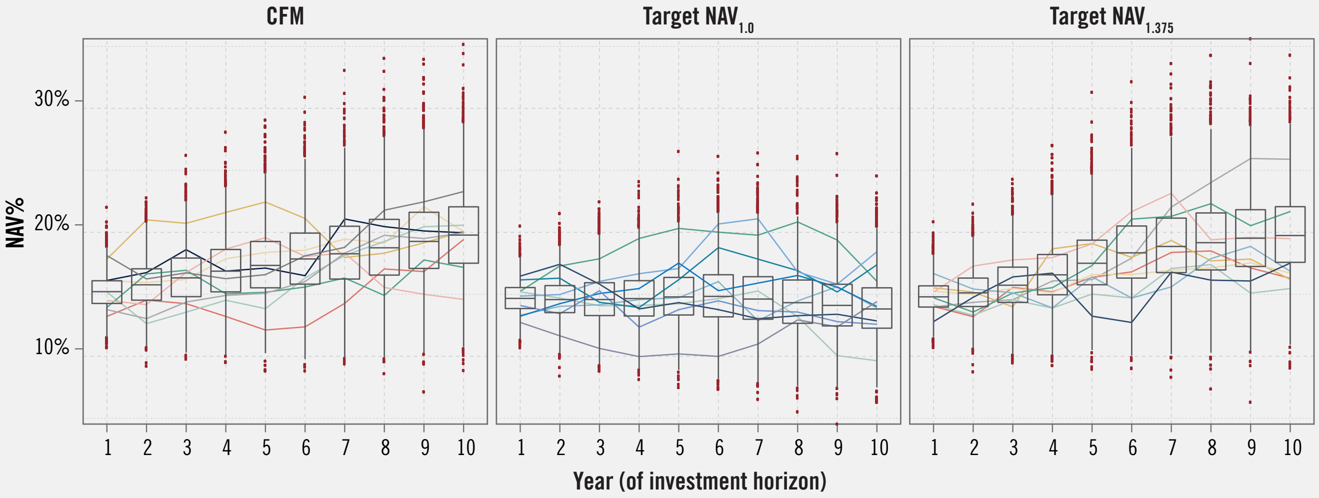
In the figures below, we choose two values for f: $f = 1.0$ and $f = 1.375$ for the Target NAV commitment strategy (*i.e.*, Target NAV$_{1.0}$ and Target NAV$_{1.375}$ respectively). $f = 1.0$ is chosen to maintain the NAV% around its initial value, while $f = 1.375$ (found by trial-and-error) is chosen so that the ending NAV% of the overall portfolio matches the ending NAV% of the CFM strategy. This facilitates direct comparison of these two strategies.

Figure 5 shows the growth of NAV as a percentage of the overall portfolio over the next 10y. Also shown are nine randomly chosen paths (from the 5,000 simulation runs). For the Target NAV$_{1.0}$ strategy the NAV percentage of the total portfolio is stable at around 15% (as intended) while the NAV percentage increases steadily for the other two strategies, CFM and Target NAV$_{1.375}$, both reaching 20% at the end of the investment horizon. This demonstrates the ability of the Target NAV strategy, as its name implies, to control or target the NAV% of the portfolio overtime.

In contrast, the CFM strategy does not control NAV growth over time. Instead, it explicitly tries to manage the net cash flow from the private asset portfolio to have minimal impact on the remaining public portfolio. Figure 6 presents the resulting net cash flows (aggregated over a year) over the next 10y. As expected, the mean of net cash flows of the CFM strategy stays close to zero, while the Target NAV$_{1.375}$ strategy results in more negative net cash flows in the middle part of the investment horizon. In contrast, the Target NAV$_{1.0}$ strategy, the most conservative strategy, shows more positive net cash flows throughout the 10y investment horizon because this strategy commits capital relatively conservatively compared to the other two.

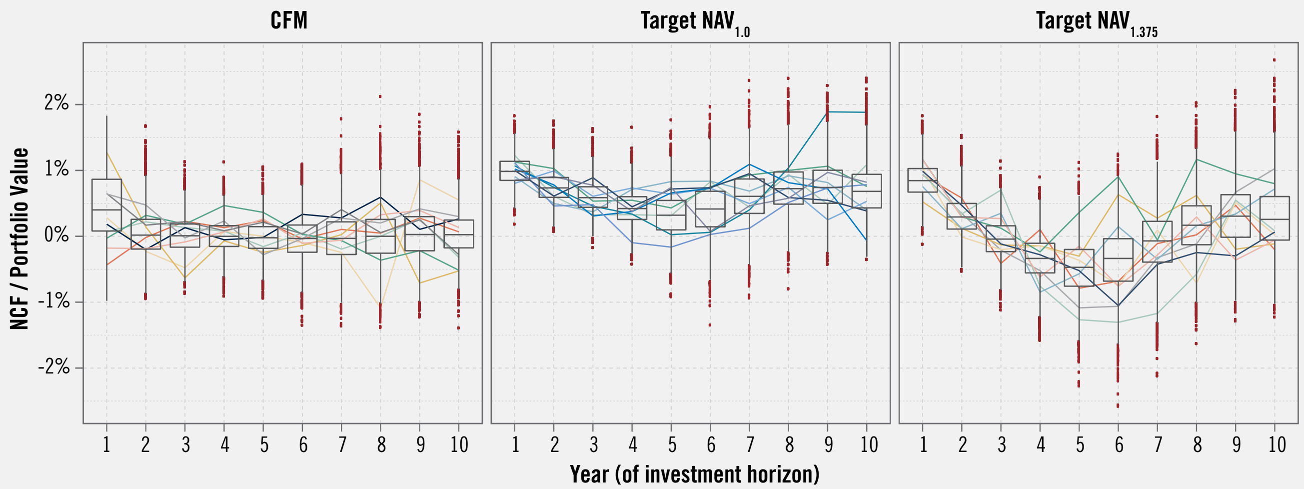
5 Appendix A2 contains the public asset capital market assumptions, the investor's views on the performance of private assets relative to public assets and their fund-selection skill used in this case study.
6 Investors can include other types of private assets, such as LP mezzanine and LP real estate funds based on their private asset allocations.

Figure 5: NAV% (as a % of End-of-Year Portfolio Value), Various Commitment Strategies



Note: The box plots display the distribution of NAV as a % of the end-of-year portfolio value based on first quartile, median and third quartile. Nine paths are shown, chosen randomly.
Source: GIC EIS & PGIM IAS. Provided for illustrative purposes only.

Figure 6: Annual Net Cash Flow (as a % of End-of-Year Portfolio Value), Various Commitment Strategies



Note: The box plots display the distribution of annual net cash flow as a % of end-of-year portfolio value based on first quartile, median and third quartile. Nine paths are shown, chosen at random.
Source: GIC EIS & PGIM IAS. Provided for illustrative purposes only.

The differences in the net cash flows from various commitment strategies have consequences for portfolio liquidity. Figure 7 shows a frequency table of different types of liquidity events (out of 5,000 simulation runs). Note, Target NAV$_{1.0}$ having the most quarterly net cash flows being positive, has only one capital call (CC) and no dry powder (DP) events. However, Target NAV$_{1.375}$ has many such events (837 1A_CC and 85 1B_CC) because of the risky nature of the strategy in terms of liquidity. Although CFM and Target NAV$_{1.375}$ have ending NAV%s that are very close, CFM manages liquidity much better than Target NAV$_{1.375}$ with only 202 1A_CC events compared to 837 1A_CC events in Target NAV$_{1.375}$.

In practice, the rebalancing target is usually subject to a window around a pre-defined allocation rather than an exact number. In addition, rebalancing may be achieved by using derivatives rather than selling physical public assets. Consequently, the number of rebalancing liquidity events might be significantly reduced and might have less severe liquidity impact.

Figure 7: Performance of Commitment Strategies: Frequency of Different Types of Liquidity Events

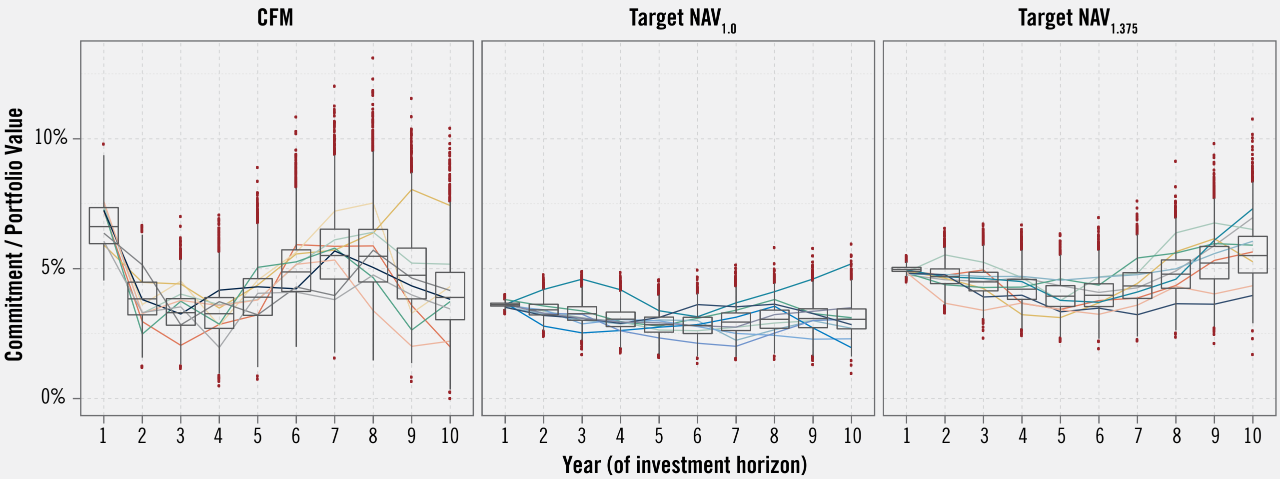
Liquidity Events	CFM	Target NAV$_{1.0}$	Target NAV$_{1.375}$
1B_RB	4,969	4,969	4,969
1A_CC	202	1	837
1B_CC	1	0	85
2_CC	0	0	0
1B_DP	0	0	0
1B_DP_RB	1,798	1,798	1,803
No Event	28	29	7

Note: RB = rebalancing; CC = capital commitment; DP = dry powder.
Source: GIC EIS & PGIM IAS. Provided for illustrative purposes only.

Figure 8 presents the commitment amount (aggregated over a year) as a percentage of the overall portfolio over the next 10y. CFM displays a cyclical and volatile commitment pattern due to its reactive nature. When a large distribution is expected in the next two quarters, due to a higher NAV valuation in the current quarter, the strategy will make larger commitments. These large commitments will subsequently generate larger capital calls which, in turn, will slow down future commitments. But, corresponding to the large capital calls, large distributions will eventually be returned resulting in cyclical commitment rallies and slowdowns.

Target NAV is not as reactive as CFM because it operates on known, not predicted, information and, as a result, it has a smoother commitment pattern. Compared with Target NAV$_{1.0}$ which has stable commitment pattern, commitments for Target NAV$_{1.375}$ grow but are still not as volatile as CFM.

Figure 8: Commitment Amount (as a % of End-of-Year Portfolio Value), Various Commitment Strategies



Note: The box plots display the distribution of commitment amount as a % of end-of-year portfolio value based on first quartile, median and third quartile. Nine paths are shown, chosen at random.
Source: GIC EIS & PGIM IAS. Provided for illustrative purposes only.

Figure 9 compares the portfolio's performance under different commitment strategies. Target NAV$_{1.0}$, as the most conservative strategy, has the lowest average portfolio horizon return (10.3%) and average horizon NAV growth (9.3%) compared with the other two strategies. Comparing CFM with Target NAV$_{1.375}$ produces some interesting results. First, the standard deviation of horizon NAV growth is much higher under CFM (3.9%) than that under Target NAV$_{1.375}$ (3.0%), resulting from CFM's volatile commitment pattern.

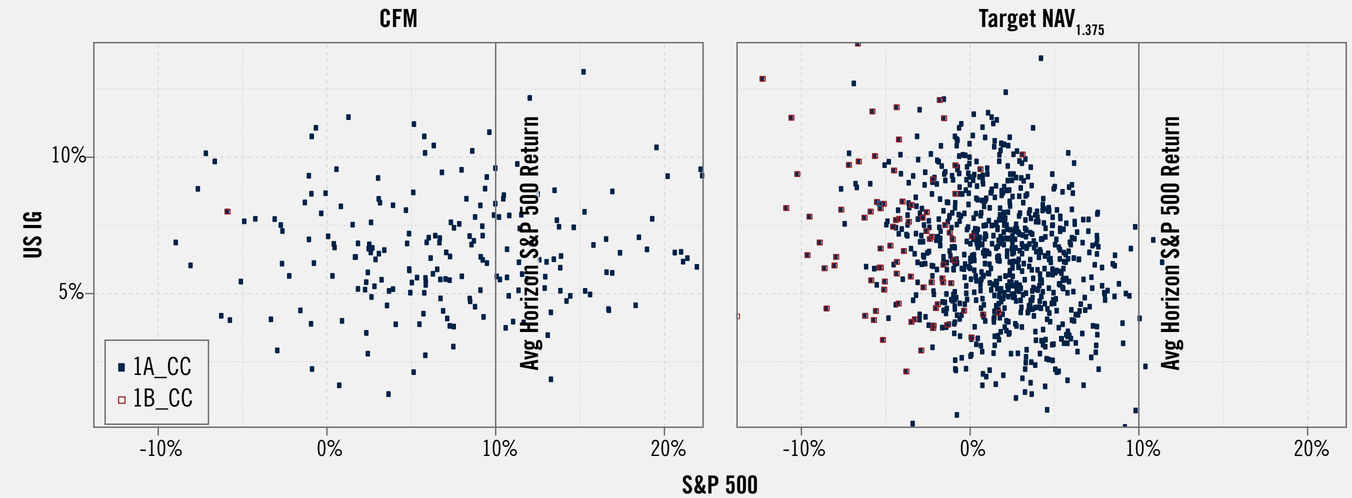
Figure 9: Performance of Commitment Strategies: Annualized Horizon Performance

Commitment Strategy	Horizon Performance				
	Port. Return (Mean)	Port. Return (Stdev)	Port. Return (Mean) (Runs with 1A_CC)	NAV Growth (Mean)	NAV Growth (Stdev)
CFM	10.4%	3.8%	9.0%	13.4%	3.9%
Target NAV$_{1.0}$	10.3%	3.8%	-2.1%[7]	9.3%	2.9%
Target NAV$_{1.375}$	10.4%	3.8%	5.5%	13.4%	3.0%

Note: For each simulation run, the portfolio horizon return is calculated as the annualized geometric mean of the horizon value multiple (*i.e.*, horizon value divided by initial value), while the horizon NAV growth rate is calculated as annualized geometric mean of the NAV multiple (*i.e.*, horizon NAV divided by initial NAV). The portfolio horizon return/NAV growth rate (mean) represents the average of all horizon returns/NAV growth rates across simulation runs. The portfolio horizon return/NAV growth rate (standard deviation) represents the standard deviation of all horizon returns/NAV growth rates across all simulation runs. 5,000 simulation runs.
Source: GIC EIS & PGIM IAS. Provided for illustrative purposes only.

Although CFM and Target NAV$_{1.375}$ have similar horizon returns, the average horizon return for those simulation runs that encounter 1A_CC events is much higher under CFM (9.0%) than that under Target NAV$_{1.375}$ (5.5%). This difference arises from the more severe public market environments in which 1A_CCs occur under Target NAV$_{1.375}$ compared to CFM. Under CFM, 1A_CCs could happen regardless of the market environment (Figure 10). However, under Target NAV$_{1.375}$, all 1A_CCs occur when the market performs below average suggesting that capital call liquidity shortages occur when investor's may most need liquidity. Since CFM tries to produce a zero net cash flow irrespective of the market environment, investors who target a higher horizon NAV%, and also want to avoid liquidity events in a bad economy, might wish to favor CFM over Target NAV$_{1.375}$.

Figure 10: Liquidity Event Sensitivity to Market Environment

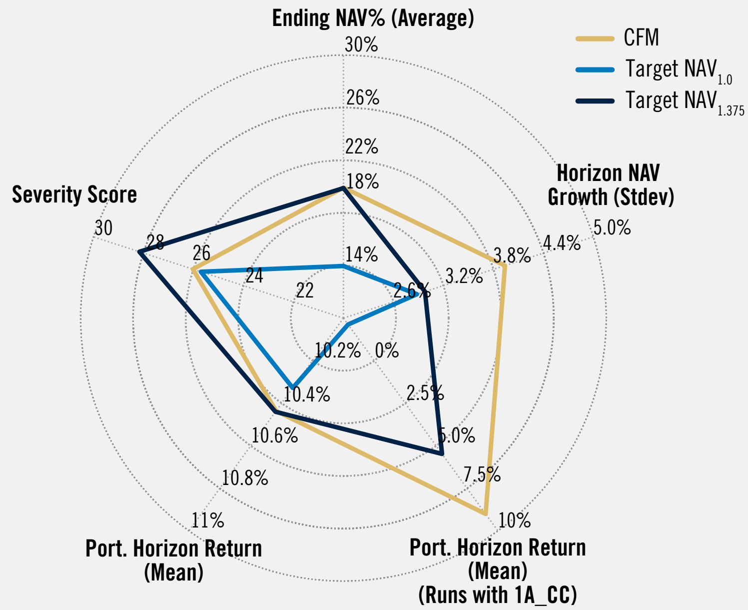


Source: GIC EIS & PGIM IAS. Provided for illustrative purposes only.

Figure 11 compares the three commitment strategies. The investor can select the appropriate strategy to match their objectives. While Target NAV$_{1.0}$ is suitable for maintaining an existing private allocation, CFM or Target NAV$_{1.375}$ may be more suitable for building up an allocation. Of the latter two, CFM may be better suited if liquidity is an investor's major concern. Otherwise, if a stable commitment pattern and / or low dispersion of horizon NAV growth is more desirable, then Target NAV$_{1.375}$ may be the better choice.

7 The Portfolio Return (Mean) (Runs with 1A_CC) of -2.1% under Target NAV$_{1.0}$ is the portfolio horizon return of the one (and only) simulation run that encounters a 1A_CC liquidity event.

Figure 11: Comparison of Commitment Strategies
Interaction of Commitment Strategy, Portfolio Performance and Liquidity



Source: GIC EIS & PGIM IAS. Provided for illustrative purposes only.

Finding the Desired Allocation to Private Assets

Having shown that a portfolio's asset allocation and commitment strategy simultaneously determine a portfolio's expected performance and liquidity, we can address the second question for investors in private assets:

2) What should be the **desired allocations** (public *vs.* private, public passive *vs.* public active) given the investor's liquidity risk tolerance?

To address this, we first fix a commitment strategy (Target $NAV_{1.375}$), and then examine how a portfolio's asset allocation affects its performance and liquidity.

Figure 12 shows different initial portfolio allocations (in terms of their deviation from the case study's baseline allocation). Portfolios along the horizontal axis from left to right increase the allocation to private assets by lowering the allocation to public passive assets. Portfolios along the vertical axis from bottom to top increase the allocation to public active alpha assets also by lowering the allocation to public passive assets. For example, to move from the baseline portfolio allocation (at the origin) to p5 (the portfolio to its left), 2% of the overall portfolio allocation is shifted from private to public passive assets.

Figure 13 shows the performance-liquidity risk tradeoffs for all portfolios in Figure 12. The dotted line represents the "efficient frontier" (corresponding to the dotted line identified in Figure 12). Given the assumptions used in the case study, the portfolios on this efficient frontier either have lower allocations to public active assets or higher allocations to private assets.

The green circle contains three portfolios with the same private allocation, with p4 (p3) having the highest (lowest) allocation to public active alpha assets. The three portfolios have similar horizon returns, but in terms of liquidity risk, as measured by the portfolio severity score, p3 is the most efficient.

The red circle contains portfolios with similar severity scores. p2, with the highest allocation to private and lowest allocation to public active, is the most efficient by generating the highest return. This shows that private assets, given the case study's assumptions, are more efficient in generating performance compared to public active assets.

Of the nine portfolios, p8 has the lowest allocation to public passive assets. Its severity score increases significantly from its neighbor p6 on the efficient frontier but not much portfolio return is added. This follows from our assumption for the liquidity event severity values (Figure 3) and the fact that p8 has just a 1% allocation to public passive assets. The portfolio liquidity severity score reflects both how much liquidity a certain asset allocation choice can support and the investor's subjective view on the severity of a given liquidity event. Regardless, this result indicates the importance of the public passive allocation for liquidity purposes.

Figure 12: Different Portfolio Asset Allocations

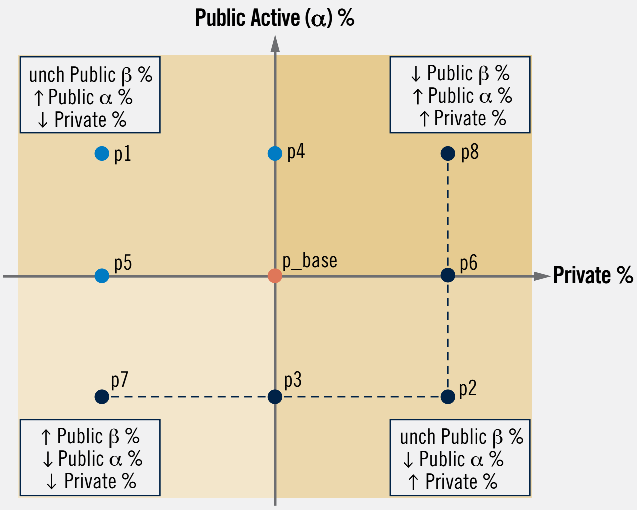
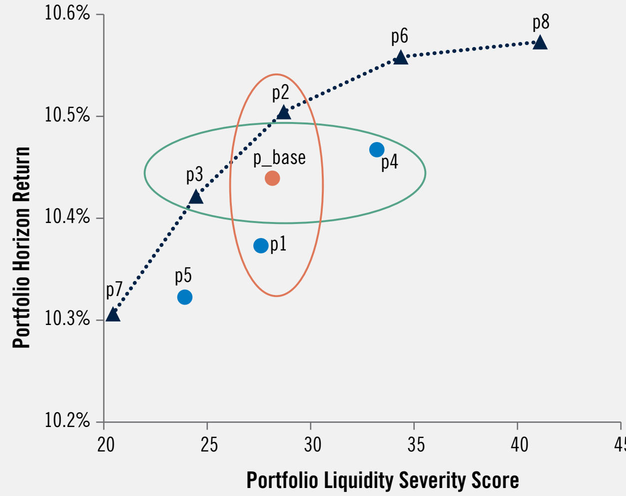


Figure 13: Asset Allocation and Performance-Liquidity Risk Tradeoff



Source: GIC EIS & PGIM IAS. Provided for illustrative purposes only.

Subject to the investor's specification of liquidity event severity values, their public asset (passive and active) risk and return assumptions, views on private assets performance and fund-selection skill, an investor can form a customized efficient frontier, on which they could move along to choose appropriate asset allocations – both public and private – given their desired level of portfolio liquidity risk.[8]

Scenario Analysis

Investors can use the framework to conduct scenario analyses to answer the final question:

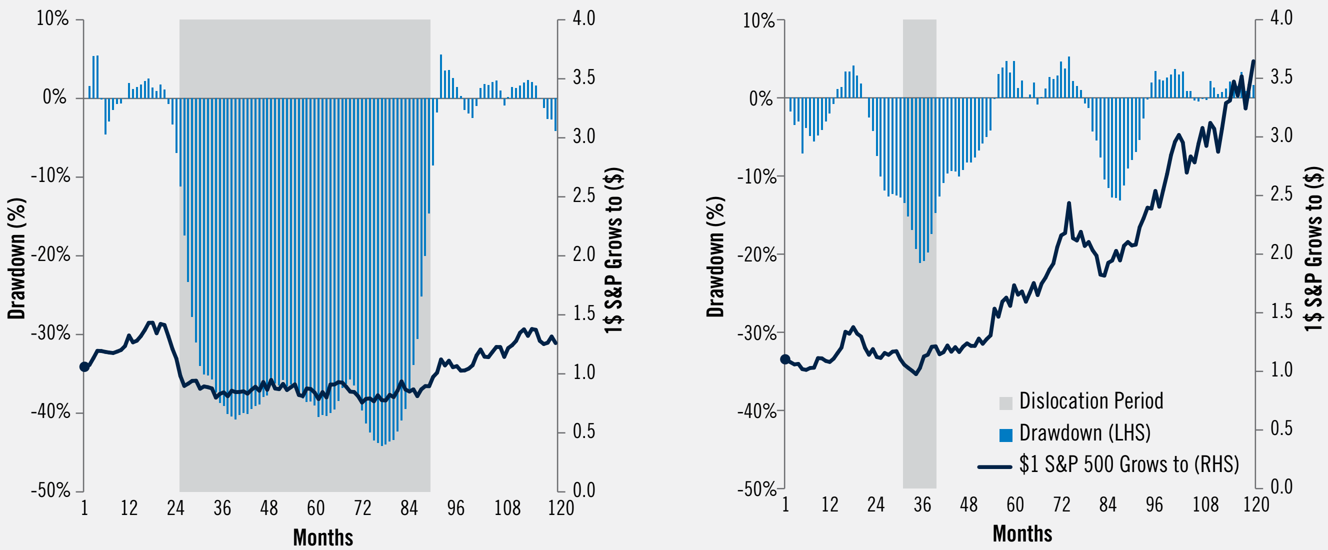
3) How would various **market scenarios** impact the portfolio's liquidity and performance?

For example, suppose an investor is interested in their portfolio's liquidity characteristics during and following a large equity market drawdown (say, -15%), given their asset allocation and private asset commitment strategy. We define two types of scenarios: a **U-shape recovery** (slow recovery) when the bad economic period lasts more than 4y and a **V-shape recovery** (quick recovery) when the bad economic period is relatively brief, lasting less than 1y. Figure 14 shows an example of both recoveries.

Figure 15 shows how rebalancing and capital call liquidity shortages arise when a U-shape scenario occurs. Before entering the dislocation (or "bad" economy) period (*i.e.*, the grey shaded area), stock prices rise faster than bond prices, leading to quick exhaustion of liquid passive stocks for rebalancing purposes. As a result, Level 1B Rebalancing Liquidity Shortages (green circles) are flagged until stock prices decline. During the period of market dislocation, private asset distributions are less than capital calls, and the resulting negative cash flows are funded by selling public stocks and bonds within the capital reserve (Level 1), as evidenced by gradual reduction of the orange-dotted line. Such depletion ultimately produces Level 1A Capital Call Liquidity Shortages (orange triangles).

8 A higher allocation to private assets leads to more volatile horizon portfolio returns. This risk measure can be included in this framework. Specifically, risk-adjusted portfolio horizon returns can be plotted on the vertical axis in lieu of portfolio horizon returns. In addition, investors can replace the liquidity risk measure by adjusting the severity values for different liquidity events or other quantitative measures such as the number of liquidity events.

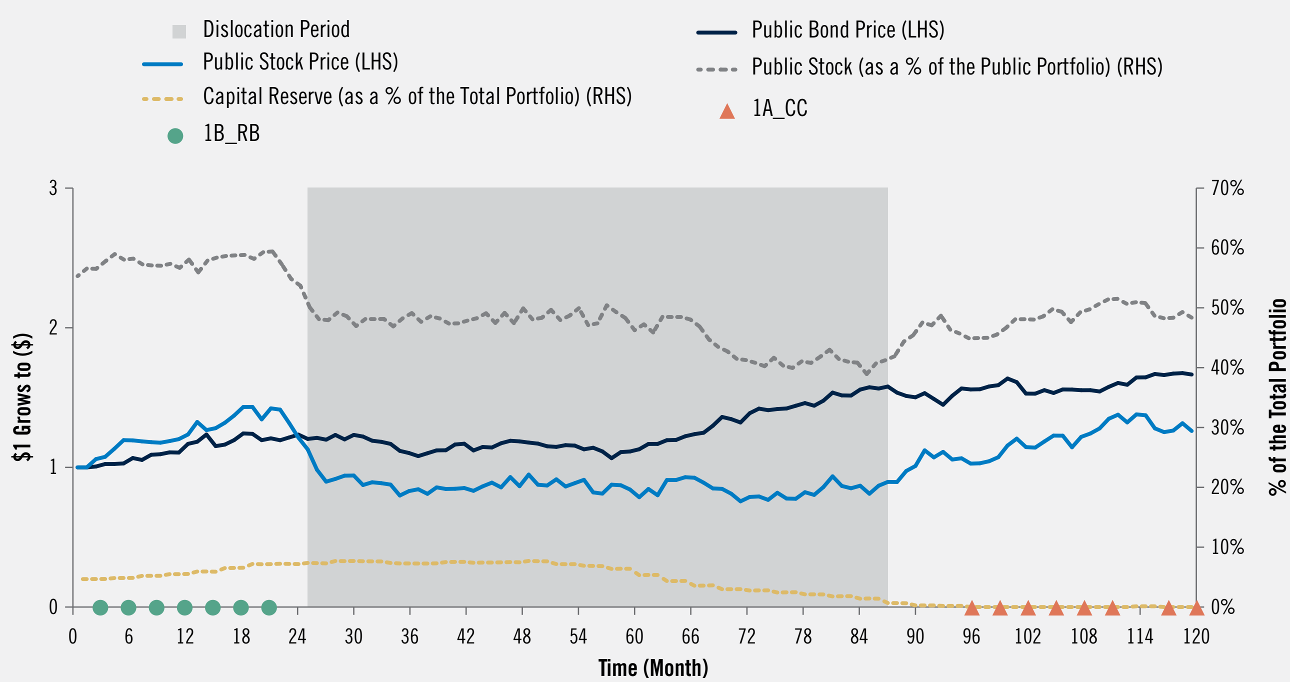
Figure 14: Severe Market Scenario Analysis



Source: GIC EIS & PGIM IAS. Provided for illustrative purposes

Figure 16 compares portfolio liquidity and performance depending on if the economic path encounters a U-shape vs. a V-shape recovery. The economic paths with one V-shape recovery leads to expected portfolio return (11.3%), much higher than the ones with U-shape recovery (6.6%). In addition, if within 10y there is a U-shape recovery, on average a capital call liquidity shortage (1A_CC) occurs 3 times over the 10y horizon, compared to no capital call event for economic paths with a V-shape recovery.

Figure 15: Liquidity Shortage Generation from U-shape Recovery



Note: The liquidity shortage generation illustration is based on the case study's baseline portfolio assumptions and Target $NAV_{1.375}$ commitment strategy.
Source: GIC EIS & PGIM IAS. Provided for illustrative purposes.

Figure 16: Comparison of Portfolio Liquidity and Performance (U-shape *vs.* V-shape)

	U-shape	V-shape
Occurrence of 1A_CC in 10y	3	0
Expected Portfolio Horizon Return	6.6%	11.3%
Number of Economic Paths (out of 5,000)	193	789

Note: The liquidity shortage generation illustration is based on the case study's baseline portfolio assumptions and Target $NAV_{1.375}$ commitment strategy.
Source: GIC EIS & PGIM IAS. Provided for illustrative purposes.

CONCLUSION

We present an asset allocation framework that models the interaction of top-down asset allocation decisions with bottom-up private asset investing and commitment strategy choice. The framework formally integrates liquidity measurement and cash flow management into a multi-asset, multi-period portfolio construction process.

The framework allows investors to find the efficient portfolio allocation given their liquidity risk tolerance. Moving onto the efficient frontier means investors could alter their top-down portfolio allocation decision to offset negative liquidity consequences from following a certain commitment strategy without sacrificing return or to reinforce positive portfolio performance without introducing additional liquidity risk.

The framework helps investors understand the interaction of their portfolio structure (*i.e.*, asset allocation between private and public assets, as well as the allocation within the public portfolio) and their choice of a private asset commitment strategy on expected portfolio performance and liquidity.

The framework is flexible and highly customizable to incorporate investors' own capital market assumptions, views on private asset performance and their fund-selection skill, private asset cash flow modeling, as well as a variety of commitment strategies. Investors may also use the framework to conduct sensitivity analysis and stress testing to evaluate how their portfolios may behave in various economic scenarios.

Acknowledgments

We wish to thank Rohan Bahri (GIC), Kevin Bong (GIC), Dr. Taimur Hyat (PGIM), Dr. Holger Dirk Michaelis (GIC) and Ashish Mittal (GIC) for their valuable contributions to this research.

REFERENCES

Buchner, A., C. Kaserer, and N. Wagner (2010). "Modeling the Cash Flow Dynamics of Private Equity Funds: Theory and Empirical Evidence." *The Journal of Alternative Investments.* **13(1).** 41–54.

Cardie, J. H., K. A. Cattanach and M. F. Kelly (2000). "How Large Should Your Commitment to Private Equity Really Be?" *The Journal of Wealth Management.* Fall **3(2).** 39–45.

de Malherbe, E. (2004). "Modeling Private Equity Funds and Private Equity Collateralized Fund Obligations." *International Journal of Theoretical and Applied Finance.* **7(3).** 193–230.

de Malherbe, E. (2005). "A Model for the Dynamics of Private Equity Funds." *The Journal of Alternative Investments.* **8(3).** 81–89.

Jeet, V. (2017). "Endurance of Internal Rate of Return." Working Paper. Burgiss Applied Research.

Kinlaw, W., M. Kritzman, D. Turkington (2013). "Liquidity and Portfolio Choice: A Unified Approach." *The Journal of Portfolio Management.* **39(2).** 19–27.

Lerner, J., A. Schoar, and W. Wongsunwai (2007). "Smart Institutions, Foolish Choices? The Limited Partner Performance Puzzle." *Journal of Finance.* **62**. 731–764.

Lo, A., C. Petrov, and M. Wierzbicki (2003). "It's 11pm—Do you Know Where Your Liquidity Is? The Mean-Variance Liquidity Frontier." *Journal of Investment Management.* 1. 55–93.

Meads, C., N. Morandi and A. Carnelli (2016). "Cash Management Strategies for Private Equity Investors." *Alternative Investment Analyst Review.* **4(4).** 31–43.

Mathonet, P.-Y. and T. Meyer (2008). "J-Curve exposure: Managing a Portfolio of Venture Capital and Private Equity Funds." John Wiley & Sons. Chap. 7. 109–113.

Robinson, D. T. and B. A. Sensoy (2016). "Cyclicality, Performance Measurement, and Cash Flow Liquidity in Private Equity." *Journal of Financial Economics.* **122(3).** 521–543.

Nevins, D., A. Conner, and G. McIntire (2004). "A Portfolio Management Approach to Determining Private Equity Commitments." *The Journal of Alternative Investments.* **6(4).** 32–46.

Oberli, A. (2015). "Private Equity Asset Allocation: How to Recommit?" *The Journal of Private Equity.* **18(2).** 9–22.

O'Shea, L. and V. Jeet (2018a). "What Happens to Cash Flows During a Crisis?" Burgiss Research Brief.

O'Shea, L. and V. Jeet (2018b). "Modeling Cash Flows for Private Capital Funds." Working Paper. Burgiss Applied Research.

O'Shea, L. and V. Jeet (2018c). "Budgeting for Capital Calls: A VaR-Inspired Approach." Working Paper. Burgiss Applied Research.

Takahashi, D. and S. Alexander (2002). "Illiquid Alternative Asset Fund Modeling." *The Journal of Portfolio Management.* **28 (2).** 90-100. DOI: https://doi.org/10.3905/jpm.2002.319836

Van Luu, B., Y. Sharaiha, N. Doskov, C. Patel and D. Turkington (2014). "The Shadow Price of Liquidity in Asset Allocation – A Case Study." *Journal of Investment Management.* **12(2).** 14–31.

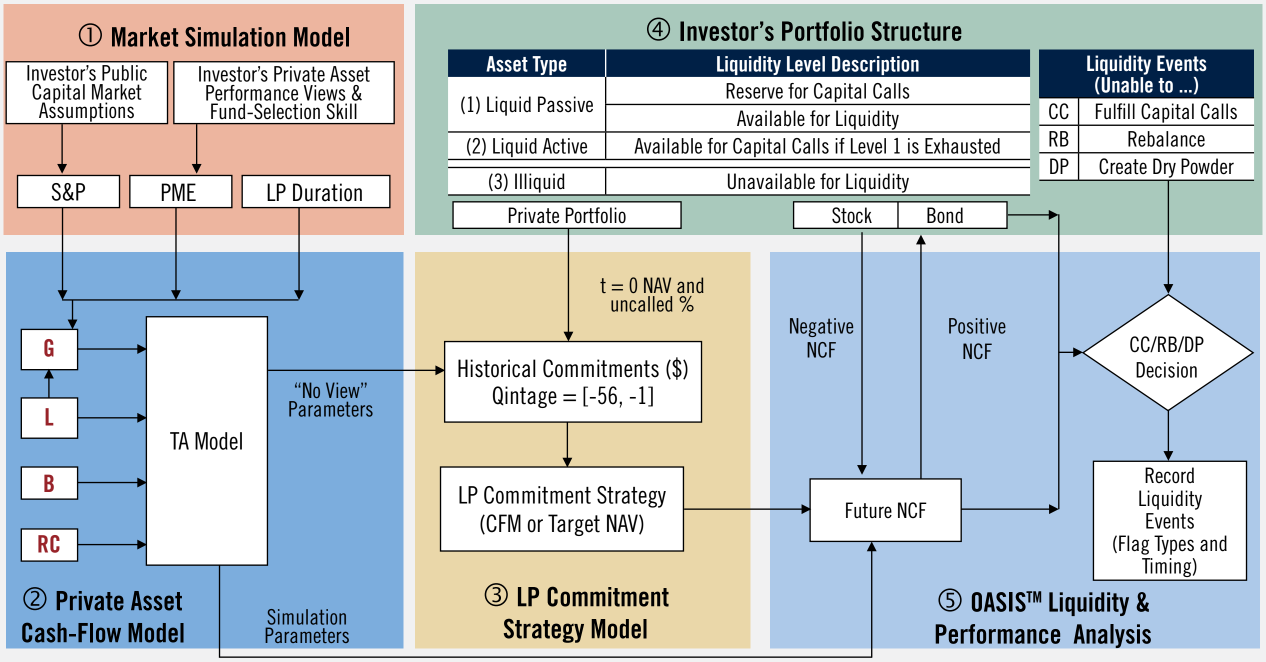
Teng, Y. and J. Shen (2019). "Asset Allocation for 'End-State' Portfolios." PGIM IAS.

Zwart, G., B. Frieser, and D. Dijk (2012). "Private Equity Recommitment Strategies for Institutional Investors." *Financial Analysts Journal.* **68(3).** 81–100.

APPENDIX

A1. Asset Allocation Framework Flowchart

Figure A1: Flowchart of the OASIS Asset Allocation Framework (Module Details)



Source: GIC EIS & PGIM IAS. Provided for illustrative purposes only.

A2. Market Simulation

Public Asset Return and Risk Assumptions

Each simulation run starts with sampling monthly market returns for both stocks and bonds.[9] Public passive assets exhibit different return and risk characteristics under different capital market environments (*i.e.*, "good" *vs.* "bad" state of economy). We define a "bad" economy when the monthly moving average (6m, backward-looking) of the S&P 500 cumulative total return experiences a drawdown of more than -15% or larger.

Figure A2 shows capital market assumptions for the two public passive assets in different economic environments. We assume public active assets generate an alpha over simulated public passive asset returns: 100bp/y for active equity strategies and 50bp/y for active fixed income strategies.

Figure A2: Public Passive Asset Return and Risk Statistics

	Bond (Bloomberg Barclays U.S. 5-10 Year Corporate Bond Index)	Equity (S&P 500 Index)
	Return Statistics in Good Economy (Bad Economy)	
Annual Expected Return	5.1% (11.6%)	13.6% (4.8%)
Annual Standard Deviation	4.8% (7.4%)	12.7% (18.6%)
	Correlation in Good Economy (Bad Economy)	
Liquid Low-Risk	1	-
Liquid High-Risk	0.28 (0.17)	1

Note: Yellow field indicates an investor input. Historical averages based on monthly data from 1995 to 2018.
Source: Barclays POINT, GIC EIS & PGIM IAS. Provided for illustrative purposes only.

9 See Teng and Shen (2019) for more details on the simulation methodology for public asset returns.

Private Asset Performance Modeling

We connect public and private performance by using regression to predict long-term (12y) PMEs of LP investments from simulated public market monthly returns, while allowing investors to express their views on private market performance (relative to public markets) and fund-selection skill:[10]

$$\log(PME_{12y}) = \alpha + \beta_1 R_{\text{S\&P 500},12y} + \beta_2 R_{\text{US HY Bond Index},12y} + \epsilon$$

Using the estimated α and βs we predict 12y PMEs from simulated annualized 12y (since the first GP capital call) horizon returns for the S&P 500 and Bloomberg Barclays US High-Yield Bond indices. The predicted PMEs from the regression represent the private asset performance of an investor with average fund-selection skill who believes that future expected private asset performance (relative to public markets) will match historical experience. We acknowledge that investors may have different views on the expected private asset performance relative to public markets and may believe their fund-selection skill differs from the average. Investors can specify quartile probabilities for expected future private performance and fund-selection skill.[11] Figure A3 presents the assumptions used in the case study.

Figure A3: Investor's View on Private Asset Performance and Fund-Selection Skill

	Investor's View on Expected Future Private Performance	Investor's Fund-Selection Skill
Quartile 1 (Q1 - highest)	25%	25%
Q2	25%	25%
Q3	25%	25%
Quartile 4 (Q4 - lowest)	25%	25%

Note: Yellow field indicates an investor input. These are probability assumptions reflecting an investor's view that the expected private asset performance (relative to public markets) will match historical experience and average fund-selection skill. The probabilities from Q1 to Q4 add to 100%. Investors may have different private asset performance views and fund-selection skill than average.
Source: GIC EIS & PGIM IAS. Provided for illustrative purposes only.

The predicted PMEs are then converted to the IRR (Internal Rate of Return) of the private investment which, in turn, is used as the simulated growth (*G*) parameter in the TA model (to be explained in A3).

$$\text{IRR} = \frac{\log(PME)}{d} + \frac{R_{S\&P\,500}}{d} \times L,$$

where d is the endurance of IRR,[12] which is assumed either to be short (3y), medium (6y), or long (10y). We assume d equal to 10y for private assets.

"*No view*" IRR refers to the IRR calculated from historical average S&P 500 and US HY annualized horizon returns (from 1995 to 2008) with a neutral view on the expected future private performance relative to public markets and average fund-selection skill. "*No view*" IRR is used for projecting cash flows under the Cash Flow Matching (CFM) commitment strategy.

10 One may argue that log(PME) already represents excess returns so the alpha and betas of this regression would be close to zero. However, this regression does produce non-trivial values of alpha and betas because the PME computation is based on a single factor (S&P 500) that implicitly assumes zero alpha and unit beta. This means log(PME) is not truly idiosyncratic in nature and may have some residual structure.
11 See Teng and Shen (2019) for details on how an investor's views are incorporated into the performance simulation.
12 See Jeet (2017).

A3. Cash Flow Modeling

The uncertainty of capital calls (both timing and magnitude) encourages investors to keep their committed, but uncalled capital, in low-return public investments that are both liquid and low risk. However, a large amount of uncalled capital invested in such investments can be a drag on portfolio returns. Uncalled capital can be more profitably managed if capital calls could be predicted to some extent. Although the uncertainty of distributions is a less severe problem, an ability to predict distributions would allow better management of uncalled capital.

A variety of cash flow models have been developed for private investments.[13] However, the Takahashi and Alexander (TA) model is still one of the most popular models because of its simplicity and effectiveness.

The TA Model

The TA model uses intuitive relationships to capture cash flow dynamics and generate cash flows that are continuous over time.[14]

The TA model has three equations to simultaneously model contributions (*i.e.*, capital calls), distributions and NAVs.

Contribution Model

The contribution model states that the capital call amount in the next period will be proportional to the uncalled capital amount at the end of the current period (UC_{t-1}):

$$C_t = UC_{t-1} \times RC(Age_{t-1}),$$

where *rate of contribution* (*RC*) is a piecewise constant function of *Age* of commitment.

Distribution Model

The distribution model states that the distribution amount in the next period will be proportional (*rate of distribution (RD)*) to the NAV that reflects the appreciation of the underlying investment by the growth rate *G*:

$$D_t = NAV_{t-1} \times (1+G) \times \mathrm{RD}(Age_{t-1}, bow, L)$$

$$\mathrm{RD} = \left(\frac{Age_{t-1}}{L}\right)^{bow}$$

G is the expected growth rate, and if the private investment develops as expected, then G equals the internal rate of return (IRR) over the specified lifespan. The *rate of distribution* is a function of *age* of commitment, *expected lifespan* of the private investment activity (*L*) and a *bow* parameter. The *bow* parameter controls the rate at which the distribution rate changes over time. The lower the bow, the faster the initial increase of the distribution and the slower the later acceleration.

NAV Model

Given the contribution and distribution models, the NAV increases as additional capital contributions are made and as underlying investments appreciate (*G*). NAV declines as distributions are made.

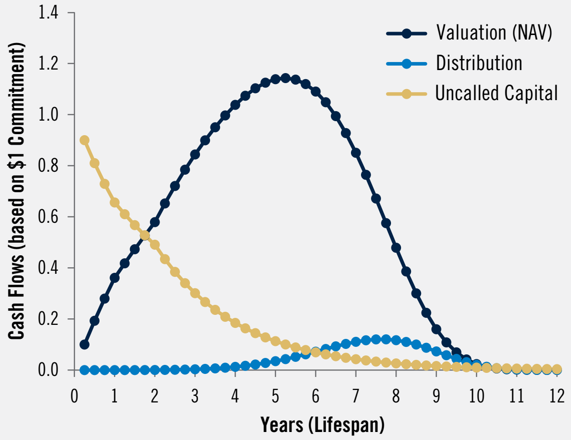
$$NAV_t = NAV_{t-1} \times (1+G) + C_t - D_t$$

Figure A4 plots the quarterly time series of uncalled capital, valuations and distributions based on the TA model for a $1 commitment using the parameter assumptions in Figure A5. The uncalled capital decays at the specified rate of contribution. The NAV initially rises and reaches a peak, and thereafter distributions are received causing valuations to decline. By the end of 12th year all cash flow activity ceases as the investment reaches the end of its lifespan.

13 See Takahashi and Alexander (2002), de Malherbe (2004, 2005), Buchner, *et al.* (2010), Robinson and Sensoy (2016), and O'Shea and Jeet (2018b, 2018c).

14 Quarterly fund-level cash flow data is very sporadic with 60-70% observations being zero, which means in most quarters nothing happens. The data get better with either temporal or cross-sectional aggregation. For our analysis we have access only to quarterly data aggregated across all funds. These data do not have any zeros, and this is when the TA model is very useful because it does not even try to model zeros.

Figure A4:
Illustrative TA Model
Cash Flows and Valuation



Source: GIC EIS & PGIM IAS. Provided for illustrative purposes only.

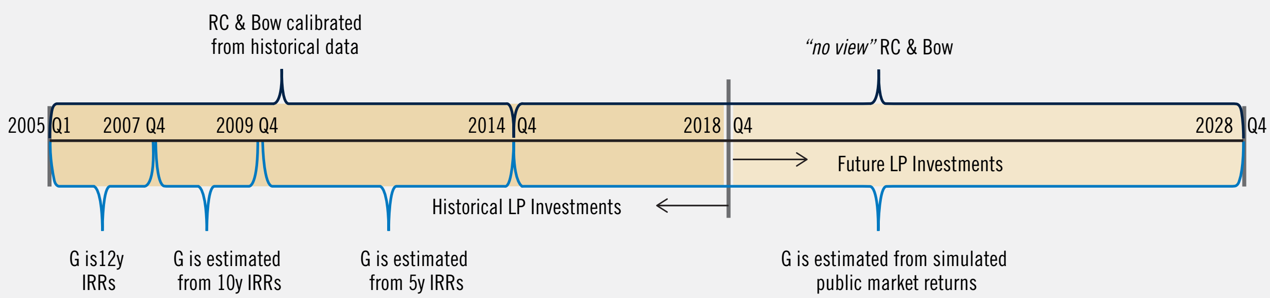
Figure A5:
TA Parameter Assumptions
(An Example)

TA Parameter		Value
RC (Age) (Quarterly)	< 4q	10%
	4 - 8q	7%
	> 8q	12%
Bow		4
G (Quarterly)		3%
Lifespan (Quarters)		48

TA Parameter Assumptions in OASIS

In our asset allocation framework, we assume it is the end of 2018 and the investor has been investing in private assets for the last 14y (56 quarters). The investment horizon is 10y and we look to commit to private assets every quarter for the next 10y (40 quarters). We rely on the TA model to esimate cash flows and valuations for both the historical 56 "qintages" (*i.e.*, quarterly vintages) and the future 40 qintages of LP investments with different parameter assumptions (*RC, bow, G*). We further assume that the private asset in our porfolio is US buyout and estimate parameter values using Burgiss US buyout data (cash flows, valuations and IRR). Lifespan is assumed to be 12y (or 48 quarters) for all LP investments. Figure A6 shows OASIS' TA model parameter assumptions.

Figure A6: OASIS TA Parameter Assumptions



Source: GIC EIS & PGIM IAS. Provided for illustrative purposes only.

We break the 56 historical qintages to two intervals – from 2005 to 2014 (40 qintages) and from 2015 to 2018 (16 qintages). For qintages 2005 to 2014, the cash flow and valuation observations from Burgiss are sufficient to estimate vintage-specific RCs and bows. Reported pooled vintage-level IRRs over the lifespan can be used for parameter Growth (G). From 2005 to 2007, 12y IRR is available, so we use 12y IRR for G; from 2008 to 2009, 10y IRR is available, so we use 10y IRR as an approximation for G; and from 2010 to 2014, we estimate 12y IRR (annualized) from 5y IRR (annualized) based on an empirical relationship: 12y IRR = 0.0736 + 0.5093 × 5y IRR (R^2 = 68.4%).

15 We do not have many observations for recent historical vintages (2015-2018). Therefore, we use "*no view*" TA parameters for these younger historical vintages. A set of "*no view*" TA parameters captures the average behavior of each parameter and is not influenced by any prediction, preference, or view of the future.

For historical qintages from 2015 to 2018, we use a "*no view*" assumption for RC and bow – the average value of the estimated historical vintage-specific RCs and bows.[15] G is estimated from simulated public market returns (discussed in the Market Simulation section).

The commitment amount for the historical 56 qintages is assumed to grow at a constant rate of 3%/quarter so that at the inception of the future 10y investment horizon, the LP portfolio valuation (NAV) matches the beginning portfolio NAV (15% × \$1b = \$150m in the case study). OASIS can accommodate any unique private asset commitment history, including building the private portfolio from the ground up.

For the 40 qintages of LP investments over the next 10y, similar to 2015-2018 assumptions, we use "*no view*" assumptions for RC and bow. G is again estimated from simulated public market returns.

Rate of Contribution (RC) and Bow Estimation

We estimate the vintage-specific RC parameters using regression between prior quarter uncalled capital (UC_{t-1}) and next quarter capital call amount (C_t) from Burgiss. We estimate RC in three age categories: 1) up to 1y old; 2) between 1y and 2y old; and 3) 2y and beyond. Figure A7 shows the quarterly RC estimates for vintages 1999 to 2014.

We estimate bow parameters using regression between prior quarter valuations (NAV_{t-1}) and next quarter distributions (D_t). Figure A8 shows the time series of bow estimates for vintages 1999 to 2014. A hump is observed around 2008 when distributions were delayed due to the global financial crisis, resulting in higher estimates of bow parameters.

Figure A7:
Rate of Contribution Estimates

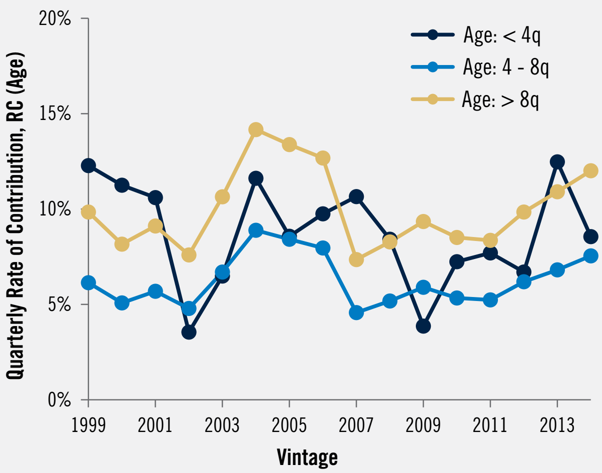
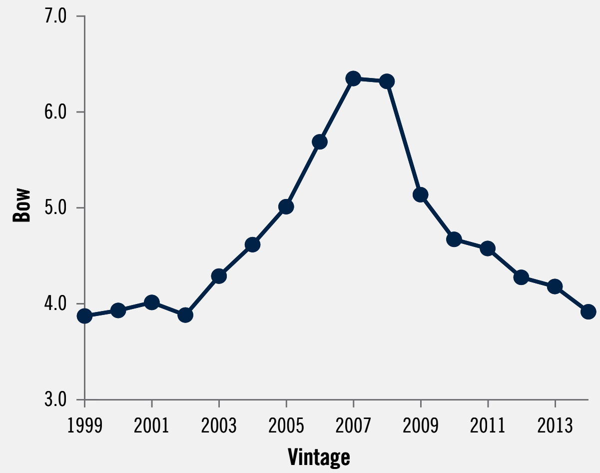


Figure A8:
Bow Estimates



Source: GIC EIS & PGIM IAS. Provided for illustrative purposes only.

OASIS does not currently model any RC and Bow dependencies on the public market. However, in scenario analysis investors can alter the RC and Bow parameter assumptions, as well as their public capital market assumptions and private asset performance views.

Figure A9 provides the values for the TA model parameter assumptions in OASIS (lifespan = 48q for all qintages).

Figure A9: RC, Bow and G Assumptions for Historical and Future LP Investments

	Quarterly Rate of Contribution (RC)			Bow	Growth
	Age: < 4q	Age: 4 - 8q	Age: > 8q		
2005	9%	8%	13%	5.0	2%
2006	10%	8%	13%	5.7	2%
2007	11%	5%	7%	6.3	3%
2008	8%	5%	8%	6.3	3%
2009	4%	6%	9%	5.1	5%
2010	7%	5%	9%	4.7	3%
2011	8%	5%	8%	4.6	4%
2012	7%	6%	10%	4.3	4%
2013	12%	7%	11%	4.2	4%
2014	9%	8%	12%	3.9	4%
2015	10%	7%	12%	4.5	Simulated
2016	10%	7%	12%	4.5	Simulated
2017	10%	7%	12%	4.5	Simulated
2018	10%	7%	12%	4.5	Simulated
2019 -2028	10%	7%	12%	4.5	Simulated

Note: Qintages 2005 to 2018 represent historical LP investments. 2019 to 2028 represent the future 40 LP investments during the 10y investment horizon.
Source: GIC EIS & PGIM IAS. Provided for illustrative purposes only.

A4. Target NAV Commitment Strategy Example

Figure A10 gives an example of how the commitment amount is determined following the Target NAV commitment strategy. We assume the total private portfolio initially consists of $150 NAV and $300 of liquid passive assets including $200 of "Committed, but Uncalled" capital (1A_a) and $100 of "Uncommitted" capital (1A_b). Since the beginning-of-the-period uncommitted capital (1A_b) is $100 and we assume $f = 0.85$, the commitment amount for this period would be $85. Subsequently 1A_b is set to $15 and the uncalled capital (1A_a) is set to $285.

Figure A10: Illustration of Target NAV Strategy, $f = 0.85$

Asset Type	Description
Liquid Passive Assets	**1A_a** Committed, but Uncalled ($200 → $285)
	1A_b Uncommitted ($100 → $15)
Illiquid Assets	NAV of Private Assets ($150)

Source: GIC EIS & PGIM IAS. Provided for illustrative purposes only.

A5. Liquidity Supply Rules and Liquidity Event Triggers

Capital Calls

Investors must satisfy GP capital calls over time based on their initially committed capital amounts. The general rule for liquidating public assets to meet a GP capital call is to sell public assets based on the private assets' pre-defined "funding mix" (*e.g.*, 40% public bonds and 60% public stocks as in the case study) from the lowest liquidity level available. If this is insufficient, then sell the most liquid public asset from the same liquidity level.

Investors follow the "waterfall" below to satisfy these capital call liquidity needs.

Step 1:

- Sell Level 1A Liquid Passive Reserve for Capital Calls – as per a pre-defined public asset funding mix;
- If the above Level 1A Liquid Passive Available for liquidity *pro rata* selling exhausts either Level 1A stocks or bonds, complete the capital call with the remaining asset in Level 1A; and
- If this is insufficient, classify the event as "*Capital Call Liquidity Shortage 1A_CC*" and continue to step 2.

Step 2:

- Sell Level 1B Liquid Passive Available for Liquidity assets *pro rata* across stocks and bonds as determined by the pre-defined public asset funding mix;
- If the above Level 1B Liquid Passive Available for liquidity *pro rata* selling exhausts either Level 1B stocks or bonds, complete the capital call with the remaining asset in Level 1B; and
- If this is insufficient, classify the event as *"Capital Call Liquidity Shortage 1B_CC"* and continue to step 3.

Step 3:

- Sell the Level 2 Liquid Active assets *pro rata* across stocks and bonds as determined by the pre-defined public asset funding mix;
- If the above Level 2 selling exhausts either Level 2 stocks or bonds, complete the capital call with the remaining asset in Level 2; and
- If this is insufficient, classify the event as *"Capital Call Liquidity Shortage 2_CC."*

Rebalancing

We assume investors set initial target policy weights by asset type in their portfolio and follow a quarterly (calendar rebalancing) schedule to rebalance public stocks and bonds (Level 1B Liquid Passive & Level 2 Liquid Active) back to their relative policy weights. We assume only Level 1B Liquid Passive Available for Liquidity assets can be used for rebalancing liquidity needs.

- Every quarter end, based on which public asset is above target weight, investors sell Level 1B Liquid Passive stocks (bonds) and rebalance the proceeds (after transaction costs) to Level 1B Liquid Passive bonds (stocks);
- If this is insufficient, classify the event as "*Rebalancing Liquidity Shortage 1B_RB.*"

"Dry Powder" Creation

Liquidity needs for creating dry powder occur when values of risky public assets (*i.e.*, stocks) are depressed during a market downturn. Investors try to maintain a certain amount of liquidity in their portfolios for a tactical move into these higher beta assets from bonds during a market downturn.

We identify a dry powder creation liquidity need at the end of each month if a simulated public equity market is exhibiting a -15% drawdown or larger (*i.e.*, a "bad" economy).

Liquidity for dry powder is provided by Level 1B Liquid Passive bonds.

- Every month, if there is a need for creating dry powder, investors sell a predefined percentage (*i.e.*, 30% in the case study) of Level 1B Liquid Passive bonds and invest the proceeds in Level 1B Liquid Passive stocks (after transaction costs);
- If this is insufficient (*i.e.*, when Level 1B Liquid Passive bonds reaches zero), classify the event as "*Dry Powder Creation Liquidity Shortage 1B_DP.*"

If the market remains in a bad economy state after a dry powder creation, there would be no calendar rebalancing between public stocks and public bonds.

"Dry Powder" Reversal

When a market recovery is observed (*i.e.*, when a drawdown is less than -5% following a recovery in equity markets), a **reversal of dry powder** is required. In other words, public bonds and public stocks (Level 1B Liquid Passive & Level 2 Liquid Active) are rebalanced back to their initial relative asset allocation weights. The implementation is the same as for calendar rebalancing described above. Note that calendar rebalancing strictly happens at quarter end, while rebalancing arising from reversal of dry powder could happen in any month whenever the market recovers from a downturn and it may not be at quarter ends.

- Every month, if a market recovery is observed, investors sell Level 1B Liquid Passive stocks (bonds) and rebalance the proceeds (after transaction costs) to Level 1B Liquid Passive bonds (stocks) based on their initial relative asset allocation weights;
- If this is insufficient, classify the event as "*Dry Powder Reversal Liquidity Shortage 1B_DP_RB.*"

A6. Portfolio Allocation Details for Asset Allocation Analysis

Figure A11: Asset Allocation Details for All Portfolios

Portfolio Assumption	1A Uncalled		1A Uncommitted		1B		2		3
	Bond	Stock	Bond	Stock	Bond	Stock	Bond	Stock	NAV
p_base	1.0%	1.0%	0.0%	0.0%	2.0%	1.0%	35.0%	45.0%	15.0%
p1	1.0%	1.0%	0.0%	0.0%	2.0%	1.0%	35.0%	47.0%	13.0%
p2	1.0%	1.0%	0.0%	0.0%	2.0%	1.0%	35.0%	43.0%	17.0%
p3	1.0%	1.0%	0.0%	0.0%	3.0%	2.0%	33.5%	44.5%	15.0%
p4	1.5%	1.5%	0.0%	0.0%	0.0%	0.0%	36.5%	45.5%	15.0%
p5	1.5%	1.5%	0.0%	0.0%	2.0%	2.0%	35.0%	45.0%	13.0%
p6	0.5%	0.5%	0.0%	0.0%	1.0%	1.0%	35.0%	45.0%	17.0%
p7	1.5%	1.5%	0.0%	0.0%	3.0%	3.0%	33.5%	44.5%	13.0%
p8	0.5%	0.5%	0.0%	0.0%	0.0%	0.0%	36.5%	45.5%	17.0%

Source: GIC EIS & PGIM IAS. Provided for illustrative purposes only.

A7. Definition of U-shape and V-shape Recoveries

The definition of U-shape or V-shape recoveries is derived from the definition of the state of the economy (see A2. Market Simulation). A U-shape recovery occurs when the "bad" economy period lasts more than 48 months, while a V-shape recovery occurs when the "bad" economy period ends within 12 months.

To compare the impact of U- and V-shape recoveries on portfolio liquidity and performance, we further constrain that there should be only one bad economy period over the investment horizon. In other words, if one simulation run contains a V-shape market recovery followed by a U-shape market recovery, such simulation run is ignored since the resulting portfolio horizon return or liquidity severity cannot be associated with a single type of market recovery.

Finally, we choose economic paths (*i.e.*, simulation runs) whose last month of the investment horizon ends with "good" economy to ensure we capture a complete picture of the economic recovery.

IMPORTANT INFORMATION

The information contained herein is collectively provided by GIC, Singapore's sovereign wealth fund, GIC Private Limited, and its affiliates (collectively, "GIC"); and PGIM, Inc and its affiliated asset management companies located in jurisdictions around the world (collectively, "PGIM"). PGIM is the principal asset management business of Prudential Financial, Inc. (PFI). PGIM, Inc. is an investment adviser registered with the US Securities and Exchange Commission. PFI of the United States is not affiliated in any manner with Prudential plc, incorporated in the United Kingdom or with Prudential Assurance Company, a subsidiary of M&G plc, incorporated in the United Kingdom.

While GIC and PGIM have collaborated for purposes of conducting research and developing this paper, GIC and PGIM are not joint venturers, affiliated in any way, or collectively providing or offering any services or products.

PGIM, the PGIM logo and Rock design are service marks, and OASIS is a trademark, of PFI and its related entities, registered in many jurisdictions worldwide.

Past performance is no guarantee or reliable indicator of future results. All investments involve risk, including the possible loss of capital. These materials are for informational or educational purposes only. In providing these materials, GIC Private Limited and its affiliates and PGIM are not acting as your fiduciary.

Alternative investments are speculative, typically highly illiquid and include a high degree of risk. Investors could lose all or a substantial amount of their investment. Alternative investments are suitable only for long-term investors willing to forego liquidity and put capital at risk for an indefinite period of time. Equities may decline in value due to both real and perceived general market, economic and industry conditions. Investing in the bond market is subject to risks, including market, interest rate, issuer, credit, inflation risk and liquidity risk. Commodities contain heightened risk, including market, political, regulatory and natural conditions and may not be suitable for all investors. The use of models to evaluate securities or securities markets based on certain assumptions concerning the interplay of market factors, may not adequately take into account certain factors and may result in a decline in the value of an investment, which could be substantial.

The analysis in the paper is based on hypothetical modeling. There is no guarantee, and no representation is being made, that an investor will or is likely to achieve profits, losses or results similar to those shown. Hypothetical or simulated performance results are provided for illustrative purposes only and have several inherent limitations. Unlike an actual performance record, simulated results do not represent actual performance and are generally prepared through the retroactive application of a model designed with the benefit of hindsight. There are frequently sharp differences between simulated results and actual results. In addition, since trades have not actually been executed, simulated results cannot account for the impact of certain market risks such as lack of liquidity. There are several other factors related to the markets in general or the implementation of any specific investment strategy, which cannot be fully accounted for in the preparation of simulated results and all of which can adversely affect actual results.

All charts contained herein were created as of the date of this presentation, unless otherwise noted. Performance results for certain charts and graphs may be limited by date ranges, as stated on the charts and graphs. Different time periods may produce different results. Charts and figures are provided for illustrative purposes and are not an indication of past or future performance of any PGIM product or GIC investment.

These materials represent the views, opinions and recommendations of the author(s) regarding the economic conditions, asset classes, securities, issuers or financial instruments referenced herein, and are subject to change without notice. Certain information contained herein has been obtained from sources that PGIM and GIC believes to be reliable; however, PGIM and GIC cannot guarantee the accuracy of such information, assure its completeness, or warrant such information will not be changed. The information contained herein is current as of the date of issuance (or such earlier date as referenced herein) and is subject to change without notice. PGIM and GIC has no obligation to update any or all of such information; nor do we make any express or implied warranties or representations as to the completeness or accuracy or accept responsibility for errors. Any forecasts, estimates and certain information contained herein are based upon proprietary research and should not be considered as investment advice or a recommendation of any particular security, strategy or investment product. These materials are not intended as an offer or solicitation with respect to the purchase or sale of any security or other financial instrument or any investment management services and should not be used as the basis for any investment decision. No liability whatsoever is accepted for any loss (whether direct, indirect, or consequential) that may arise from any use of the information contained in or derived from this report. PGIM and its affiliates and GIC may make investment decisions that are inconsistent with the recommendations or views expressed herein, including for proprietary accounts of PGIM or its affiliates or GIC. The opinions and recommendations herein do not take into account individual client circumstances, objectives, or needs and are not intended as recommendations of particular securities, financial instruments or strategies to particular clients or prospects. No determination has been made regarding the suitability of any securities, financial instruments or strategies for particular clients or prospects. For any securities or financial instruments mentioned herein, the recipient(s) of this report must make its own independent decisions.

Solely with respect to PGIM delivery of this paper, in the United Kingdom and various European Economic Area ("EEA") jurisdictions, the information is issued by PGIM Limited with registered office: Grand Buildings, 1-3 Strand, Trafalgar Square, London, WC2N 5HR. PGIM Limited is authorised and regulated by the Financial Conduct Authority of the United Kingdom (Firm Reference Number 193418) and duly passported in various jurisdictions in the EEA.

These materials are issued by PGIM Limited to persons who are professional clients or eligible counterparties for the purposes of the Financial Conduct Authority's Conduct of Business Sourcebook. In certain countries in Asia, information is presented by PGIM (Singapore) Pte. Ltd., a Singapore investment manager registered with and licensed by the Monetary Authority of Singapore. In Japan, information is presented by PGIM Japan Co. Ltd., registered investment adviser with the Japanese Financial Services Agency. In South Korea, for the information provided by PGIM information is presented by PGIM, Inc., which is licensed to provide discretionary investment management services directly to South Korean investors. In Hong Kong, information is presented by representatives of PGIM (Hong Kong) Limited, a regulated entity with the Securities and Futures Commission in Hong Kong to professional investors as defined in Part 1 of Schedule 1 of the Securities and Futures Ordinance. In Australia, this information is presented by PGIM (Australia) Pty Ltd. ("PGIM Australia") for the general information of its "wholesale" customers (as defined in the Corporations Act 2001). PGIM Australia is a representative of PGIM Limited, which is exempt from the requirement to hold an Australian Financial Services License under the Australian Corporations Act 2001 in respect of financial services. PGIM Limited is exempt by virtue of its regulation by the Financial Conduct Authority (Reg: 193418) under the laws of the United Kingdom and the application of ASIC Class Order 03/1099. The laws of the United Kingdom differ from Australian laws. Pursuant to the international adviser registration exemption in National Instrument 31-103, PGIM, Inc. is informing you of that: (1) PGIM, Inc. is not registered in Canada and relies upon an exemption from the adviser registration requirement under National Instrument 31-103; (2) PGIM, Inc.'s jurisdiction of residence is New Jersey, U.S.A.; (3) there may be difficulty enforcing legal rights against PGIM, Inc. because it is resident outside of Canada and all or substantially all of its assets may be situated outside of Canada; and (4) the name and address of the agent for service of process of PGIM, Inc. in the applicable Provinces of Canada are as follows: in Québec: Borden Ladner Gervais LLP, 1000 de La Gauchetière Street West, Suite 900 Montréal, QC H3B 5H4; in British Columbia: Borden Ladner Gervais LLP, 1200 Waterfront Centre, 200 Burrard Street, Vancouver, BC V7X 1T2; in Ontario: Borden Ladner Gervais LLP, 22 Adelaide Street West, Suite 3400, Toronto, ON M5H 4E3; in Nova Scotia: Cox & Palmer, Q.C., 1100 Purdy's Wharf Tower One, 1959 Upper Water Street, P.O. Box 2380 - Stn Central RPO, Halifax, NS B3J 3E5; in Alberta: Borden Ladner Gervais LLP, 530 Third Avenue S.W., Calgary, AB T2P R3.

IAS0326-100

For inquiries and to learn more about PGIM's investment advisory capabilities, email IAS@pgim.com or visit pgim.com/IAS.