INSTITUTIONAL ADVISORY & SOLUTIONS

ASSET ALLOCATION FOR "END-STATE" PORTFOLIOS

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In recent years many US corporate pension plans have closed and entered their "end-state." As end-state plans have become more prevalent, their special portfolio management challenges, including asset allocation, have gained attention.

Pure immunization with public fixed income assets ("hibernation") is a possible investment management strategy to try to minimize funding ratio variability. But this may not be sufficient for all end-state plans. For example, mortality risk could cause the actual cash liabilities to deviate from the estimated cash liabilities. This risk may argue for the inclusion of return-seeking assets. In addition, some plans have had good performance experience with their illiquid private assets (*e.g.*, private real estate, private equity, and private credit funds). How can a CIO evaluate the potential of these private assets remaining in their end-state portfolio?

We use our asset allocation framework (OASIS™) to help end-state investors solve for optimal asset allocations. An asset allocation solution seeks to maximize the end-state portfolio's expected horizon value while meeting future cash obligations at a desired confidence level and keeping the funded status at a target level of stability over the investment horizon. We show that private assets can play an important role in helping end-state portfolios achieve their return objectives while meeting their liquidity and funded status stability constraints.

CIOs managing end-state portfolios may impose additional constraints to address their specific concerns. For example, CIOs may impose an upper limit on their total allocation to private assets, impose a floor on the plan's funding ratio, or express views on expected private asset performance and their fund-selection skills. Making asset allocation constraints more restrictive typically implies a less risky portfolio, with a lower allocation to private assets. The OASIS framework helps CIOs measure this tradeoff between performance and their constraints, allowing them to make better business decisions.

The findings shown are derived from statistical models. Reasonable people may disagree about the appropriate model and assumptions. Models should not be relied upon to make predictions of actual future account performance. See additional disclosures.

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Introduction

Following the trend of transitioning from Defined Benefit (DB) to Defined Contribution (DC) plans, many US corporate pension plans have closed, playing a smaller role in employee retention. Such pension plans have entered their "End-State." End-state portfolios share some common characteristics. Despite being closed to new entrants, the remaining benefit payments may stretch for decades. They also usually have a heavy and increasing percentage of retirees. Liabilities are either in or soon will enter a "run-off" mode (*i.e.*, periodic cash obligations decline over time).

End-state portfolios share some characteristics with ongoing ("open") plans. For example, they share common pension risks, including, but not limited to, interest rate risk, investment risk and mortality risk. Lower interest rates will result in a higher present value of pension liabilities. If investment returns are less than expected, pension sponsors may need to make additional contributions to maintain their ability to pay future pension liabilities. Furthermore, if the participants live longer than expected, pension liabilities will exceed current projections. All these risks affect whether an end-state portfolio will have sufficient assets to pay its liabilities.

Compared to a typical open-plan portfolio, end-state portfolios usually have a more certain cash liability schedule and may expect minimum (even zero) future contributions. There is a view that these portfolios should be managed at a lower risk level. However, chief investment officers (CIOs) managing end-state portfolios may still wish to carefully monitor the evolution of such portfolios in different economic environments and pursue a more active portfolio management strategy to achieve various business goals.

CIOs managing end-state portfolios may have such questions, as:

- Will a pure immunization strategy reduce my end-state portfolio risks sufficiently, given mortality uncertainty?
- How should I allocate my public and private assets if I want to achieve higher returns than investing solely in public fixed income assets?
- Do I want to use my portfolio surplus to cover additional pension obligations as part of my company's M&A strategy or fund Other Post-Employment Benefits (OPEB)[1]?
- My illiquid private assets have performed well. Is there a continued role for these assets now that the portfolio has entered "end-state"?

PGIM IAS has developed an asset allocation framework (OASIS™ – **O**ptimal **As**set Allocation with **I**lliquid Asset**s**) that can help CIOs solve for their portfolios' asset allocation, including asset allocation for end-state portfolios.[2] OASIS follows a cash flow-driven investment strategy that seeks the asset allocation that will likely enable the investor to satisfy all future cash flow liabilities with a desired level of confidence over the entire investment horizon while maximizing expected horizon value. The application of OASIS can incorporate a constraint to keep the funded status stable enough over the entire investment horizon. OASIS also incorporates the performance of common private assets (limited partnerships, or LPs) and recognizes their unique characteristics such as possible delays for the first capital call and higher and lumpier transaction costs relative to public assets. Investors can use OASIS to determine their optimal allocation across both public and private assets, and the sensitivity of their allocation to changes in assumptions about the risk and return characteristics of both public and private assets.

Some CIOs managing end-state portfolios may have the business objective of eventually offloading the pension risks to an insurance company through a Pension Risk Transfer (PRT) buyout transaction.[3] Another group of CIOs may prefer to follow a liability-driven investment (LDI) strategy that aims to achieve the same risk reduction as a PRT buyout by selling riskier assets and buying bonds that match the interest rate risk of the pension plan's liabilities, *i.e.*, immunization (hibernation) with public fixed income assets.[4] These CIOs can still use OASIS to explore asset allocations for their portfolios to gain insights on portfolio dynamics.

OASIS is flexible and customizable. CIOs can use OASIS to conduct extensive "what-if" analyses for their special business requirements. For example, a CIO with a company considering a PRT transaction in 5 years may want to have the total allocation to private assets below a certain percentage threshold. They can do so with OAISIS by imposing an upper limit on total private asset allocation as an additional constraint. As another example, if a CIO never wants the funding ratio of their end-state portfolio to fall below a certain level, they may impose a funding ratio floor constraint.

1 Using pension surplus to fund OPEB obligations is possible but subject to specific regulatory constraints.
2 See J. Shen and B. Phelps, "Illiquid Private Assets: Interaction of Illiquid and Liquid Assets in Investor Portfolios," PGIM IAS, February 2018 and J. Shen, F. Farazmand and Y. Teng "The Tradeoff between Liquidity and Performance: Private Assets in Institutional Portfolios," PGIM IAS, January 2019, for details on the framework underlying OASIS.
3 A PRT buyout transaction involves transferring the assets and liabilities of a pension plan to an insurance company. The insurance company will guarantee benefit payments to participants for life. See "Longevity and Liabilities: Bridging the Gap," PGIM, October 2016 for reference.
4 See G. Knapp "Hibernation: Managing a Sleeping Bear," PGIM Fixed Income, March 2015.

Case Study

We present a Case Study to illustrate using OASIS for US corporate end-state portfolios. We first frame the end-state portfolio asset allocation problem, including the investment objective and constraints, and then solve for the asset allocation.

We show a set of end-state portfolio asset allocation results conditional on the constraints and identify the role of private assets.[5] We also conduct sensitivity and distributional analysis of the results for a set of baseline scenarios. We then explore the impact of mortality uncertainty on portfolio asset allocation. Finally, we show a set of "what-if" analyses by adding new constraints or changing investor's views on expected private asset performance.

End-State Portfolio Asset Allocation Problem

We frame the portfolio asset allocation problem by first specifying the investment objective and constraints. We assume that the investor's objective is to maximize 10y horizon expected portfolio value. Maximizing the 10y horizon expected portfolio value helps improve the likelihood that there will be enough assets to meet cash obligations beyond the 10y horizon.

We assume that the investor maximizes horizon value subject to the constraints that first, the portfolio will satisfy future cash obligations with a certain minimum likelihood, and second, the funding ratio variability of the portfolio will not exceed a maximum threshold value over the investment horizon.[6] For example, an investor may seek to maximize their end-state portfolio's expected horizon value provided that, over the entire investment horizon, (with at least 99% probability) the portfolio satisfies the cash liabilities and the average year-over-year funding ratio variability of the portfolio does not exceed 2%. We will discuss the two end-state portfolio asset allocation constraints in more detail in the next section.

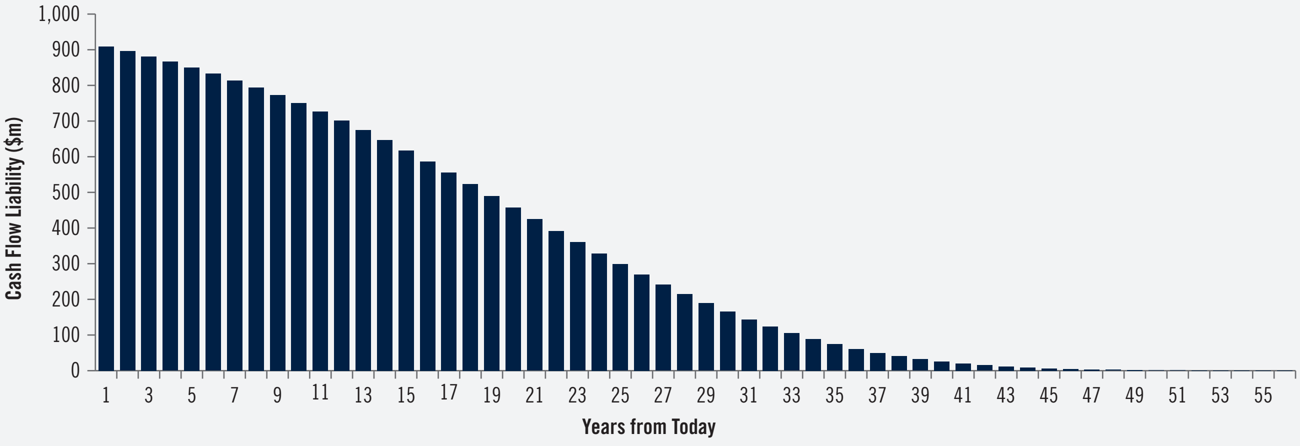
Asset Allocation Constraints

Key Constraints: Liquidity and Funded Status Stability Requirements

Liquidity Requirement

Investors specify both a periodic cash flow liability schedule and their minimum confidence level to meet these periodic cash obligations over the investment horizon. Figure 1 shows the stylized cash flow liability schedule of an end-state liability profile with a duration of around 9.5. It is observed that starting from today, the expected periodic cash liability declines steadily over time until they run out at the end of year 56.

Figure 1: End-State Cash Flow Liability Schedule
(Hypothetical Retiree-Only End-State DB Plan)



Note: The figure illustrates hypothetical yearly liability cash flow obligations for an end-state portfolio for the entire life of the pension plan. This is representative of a typical end-state portfolio of pure retiree cash flows. We focus on accounting liabilities in this paper.

Source: PGIM IAS. Example shown for illustrative purposes only.

5 Key assumptions for the investor inputs of OASIS can be found in Appendix A1: "Summary of Case Study Assumptions."

6 See Appendix A2: "Formulating End-State Portfolio Investment Objective and Constraints."

Funded Status Stability Requirement

End-state investors worry about the funded status of their portfolios. The **funded status** of a pension plan is the difference between the market value of the portfolio's assets and the present value of all future liabilities. A closely related measure is the **funding ratio**, which is the ratio of a plan's market value of assets to its present value of liabilities. A funding ratio above 100% indicates that the portfolio is able to cover all cash flow obligations (under current cash flow and discount rate assumptions) while a funding ratio below 100% indicates greater risk of possibly not being able to do so.

The funding ratio can be volatile for many reasons. For example, if not fully immunized, declining interest rates can cause the present value of the liabilities to increase and lower the funding ratio. The funding ratio can also vary depending on the economic environment, which may change the characteristics (*e.g.*, volatilities, correlations and the idiosyncratic risks) of investment returns. For example, during a recession the market value of the portfolio's assets can decrease significantly, reducing the funding ratio.

End-state investors often seek to maintain a certain level of stability of the funded status over time. We use a maximum funding ratio variability threshold to represent end-state investors' funded status stability requirement. This constraint allows investors to control the variability of their funding ratio. To calculate the funding ratio variability, we first calculate the absolute value of year-over-year funding ratio changes every month over the investment horizon (*i.e.,* a total of 108 absolute values over 120 months). Then we take the average of these absolute values.[7] Intuitively, a lower funding ratio variability threshold represents a stricter constraint on funded status stability, *e.g.,* a 2% funding ratio variability threshold is more restrictive than a 6% threshold.

Given the definitions above, calculating a portfolio's monthly funding ratio requires calculating the market value of the portfolio's assets and discounting its projected liability cash flows using a pension liability discount rate. OASIS dynamically simulates a pension discount rate based on capital market performance.[8] For each month, over the 10y investment horizon, the simulated pension liability discount rate discounts all remaining liabilities to calculate the present value of liabilities. This present value of liabilities and the simulated portfolio asset values are used to calculate the monthly funding ratio.

Other Potential Constraints

OASIS allows investors to incorporate additional constraints. For example, some end-state plan sponsors may, at some point, wish to transfer rather than manage their pension risk. There are two popular actions they may take – either transfer the risk to the participants by offering lump-sum options or transfer the risk to an insurance company via a Pension Risk Transfer (PRT) transaction (*e.g,* a "buyout" or a "buy-in"). A sponsor considering such a transfer would likely worry about having too large an allocation to private assets which may either constrain liquidity needed for offering lump-sum payments or may make a PRT more difficult since insurance companies may apply a "haircut" to the net asset value of private assets. Consequently, to address these concerns such sponsors may want to add a constraint on the maximum amount of private assets in the portfolio.

Investment Opportunity Set and Asset Dynamics

We assume there are five assets in the investment opportunity set: two public assets, including a public "low-risk" asset and a "high-risk" equity asset (*e.g.,* S&P 500); and three private assets including LP buyout private equity, LP mezzanine debt and LP real estate funds. Investors may specify certain parameters related to the risk and return characteristics of the public high-risk asset and express views on how expected future private asset performance (relative to public markets) might differ from historical experience at the private asset class level. Investors may also express views on their fund-selection skills. OASIS models public market asset dynamics conditional on the economic environment (*i.e.,* "good" or "bad" state of the economy).[9] [10]

OASIS is a high-level asset allocation tool. There are only two public assets. In reality a plan will hold many different public assets. In particular, end-state plans may hold a diverse mix of public low-risk assets (a "hedging asset portfolio") to match the duration, convexity and credit risk of the liabilities. Therefore, we define the OASIS low-risk asset to be a fixed-income "hedging asset." This asset is meant to proxy a plan's actual hedging asset portfolio constructed to constrain the plan's funding ratio variability by tracking liability duration and convexity, with full flexibility to select and adjust individual securities to boost performance and limit credit migration. In practice, the construction of a hedging asset portfolio is a challenging exercise and is typically actively managed. Nevertheless, OASIS summarizes the hedging asset portfolio with a single asset that is duration- and convexity-matched to the liabilities and whose monthly return is tied to the change in the liability discount rate (*i.e.,* AA-corporate bond yields).[11]

7 A similar and commonly used concept "funded status volatility" is defined as the average year-to-year (absolute value) percentage change in funded status (or funding ratio). Definition and analysis of this measure maybe find in "Assessing Funded Status Volatility – Pension Finance Watch: Special Analysis," Towers Watson, 2011. We use a slightly varied concept of "funding ratio variability" to monitor the year-to-year funding ratio change more frequently, *i.e.,* monthly instead of yearly.

8 Modeling details of the pension liability discount rate can be found in Appendix A3: "Modeling Pension Liability Discount Rate."

9 Public asset performance modeling details can be found in Appendix A4: "Modeling Public Asset Performance."

10 Public and private (LP allocation) horizon value distributions and correlations can be found in Appendix A5: "Public and Private Asset Horizon Value Distributions & Correlations" to help better understand the performance dynamics of the investment opportunity set and their impact on the asset allocation results.

11 The OASIS setup can allow an investor to specify how well they wish the hedging asset to actually track the performance of a perfectly constructed hedging asset portfolio. For example, the hedging asset can lag in matching duration and/or experience losses due to credit migration.

Asset Allocation Results for End-State Portfolios

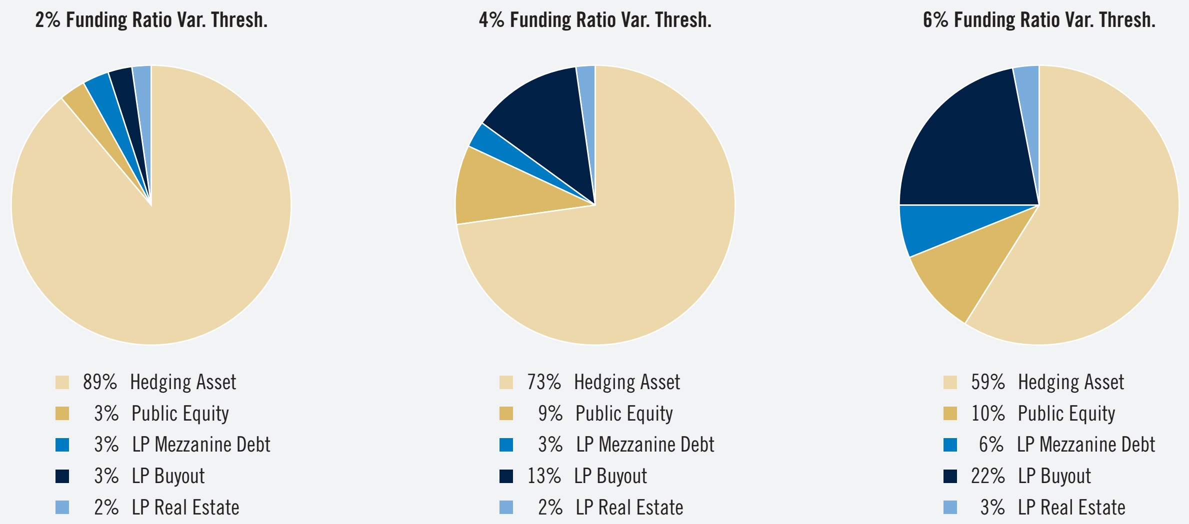
The Case Study's baseline scenarios assume an initial portfolio market value of $10,000m and a present value of all liabilities of $11,772m, based on a flat 3.9% discount rate. These assumptions produce an initial funding ratio of about 85%. The investor specifies a confidence level to meet the cash flow liability schedule in Figure 1. We assume there are no future cash contributions. OASIS produces a set of baseline asset allocation results that maximize the expected horizon value of the portfolio, subject to the two constraints: first, the portfolio has sufficient assets to meet all end-state cash liabilities with a desired minimum confidence level over the entire investment horizon; and second, the funding ratio variability of the portfolio does not exceed a specified maximum level.

Given the plan's assumed initial funding ratio of 85%, the probability of fulfilling all cash obligations in the first 10y (*i.e.*, the investment horizon) of the total of 56 years is very high. In other words, the confidence levels for many potential asset allocations are near 100%. Therefore, when we discuss results, we focus on the impact of the funded status variability constraint while keeping the liquidity requirement at a 99% confidence level.

Optimal Asset Allocation Results for Baseline Scenarios

Figure 2: Optimal Asset Allocation for End-State Portfolios
(Baseline Scenarios)

	Asset Allocation Result		
Confidence Level	**99%**	**99%**	**99%**
Funding Ratio Var. Thresh.	**2%**	**4%**	**6%**
Hedging Asset	89%	73%	59%
Public Equity	3%	9%	10%
LP Mezz. Debt	3%	3%	6%
LP Buyout	3%	13%	22%
LP Real Estate	2%	2%	3%
Average Horizon value ($m)	**6,479.5**	**7,805.7**	**8,931.8**
Horizon Value Volatility ($m)	**1,945.1**	**2,597.8**	**3,489.5**



Note: Horizon value volatility is the standard deviation of horizon values across all the successful runs for the optimal asset allocation result.

Source: PGIM IAS. Provided for illustrative purposes only.

Figure 2 shows a set of optimal asset allocation solutions corresponding to three funding ratio variability thresholds (2%, 4% and 6%). The asset allocation changes as this constraint changes. For example, at a 2% threshold, the vast majority (92%) of assets is allocated to the public portfolio: 89% is the hedging asset and 3% is public equity. Total allocation to private assets is 8% with 3% LP buyout, 3% LP mezzanine debt, and 2% LP real estate. Loosening the funding ratio variability threshold to 6% (*i.e.,* allowing the portfolio asset value to exhibit more volatility relative to liabilities) reduces the public portfolio allocation to 69%, giving more room for allocation to private assets. The 31% in the private portfolio is now composed of 22% LP buyout, 6% LP mezzanine debt, and 3% LP real estate.

A 2% threshold is a restrictive investor requirement. Nevertheless, under this constraint, there is still a meaningful amount of allocation to private assets (8%). This highlights the importance of overall private assets in balancing end-state investor's constraints and portfolio performance. The increased allocation to private assets occurs at the expense of public assets. As the funding ratio variability threshold increases (*i.e.,* becomes less strictive), the allocation to riskier and potentially more rewarding assets increases, resulting in higher expected portfolio horizon values.

Measuring the Cost of Constraints

To examine the tradeoff between portfolio performance and investor constraints we analyze the portfolio's performance sensitivity to the investor's specified funding ratio variability threshold.

Figure 3 illustrates how the asset allocation and the corresponding portfolio horizon value change with the threshold. Starting from a relatively high threshold, as the threshold decreases (*i.e.,* constraint becomes tighter), the total allocation to private assets decreases and the allocation to the hedging asset increases. For example, moving from a 6% to a 2% threshold decreases the allocation to private assets from 31% to 8%, while the allocation to the hedging asset increases from 59% to 89%. This shift to the hedging asset decreases the expected portfolio horizon value by 27%, from $8,932m to $6,479m.

The consequence of a tighter funded status stability requirement is a sharp reduction of portfolio horizon value. As the funding ratio variability threshold becomes more restrictive the horizon value decreases, with steeper drops at lower funding ratio variability thresholds. This loss in horizon value as the funded status variability threshold becomes more restrictive captures the "cost of constraints." CIOs evaluate this tradeoff between funding ratio variability threshold and portfolio performance to help make the best decisions for their end-state plans.

Distributional Analysis of Baseline Results

OASIS is simulation-based. For each potential asset allocation solution, OASIS simulates many scenarios (5,000 in the Case Study) representing asset performance in various economic environments to identify the optimal asset allocation. Simulation permits examination of the distributional characteristics of the asset allocation results to gain better understanding.

Portfolio Horizon Value Distribution

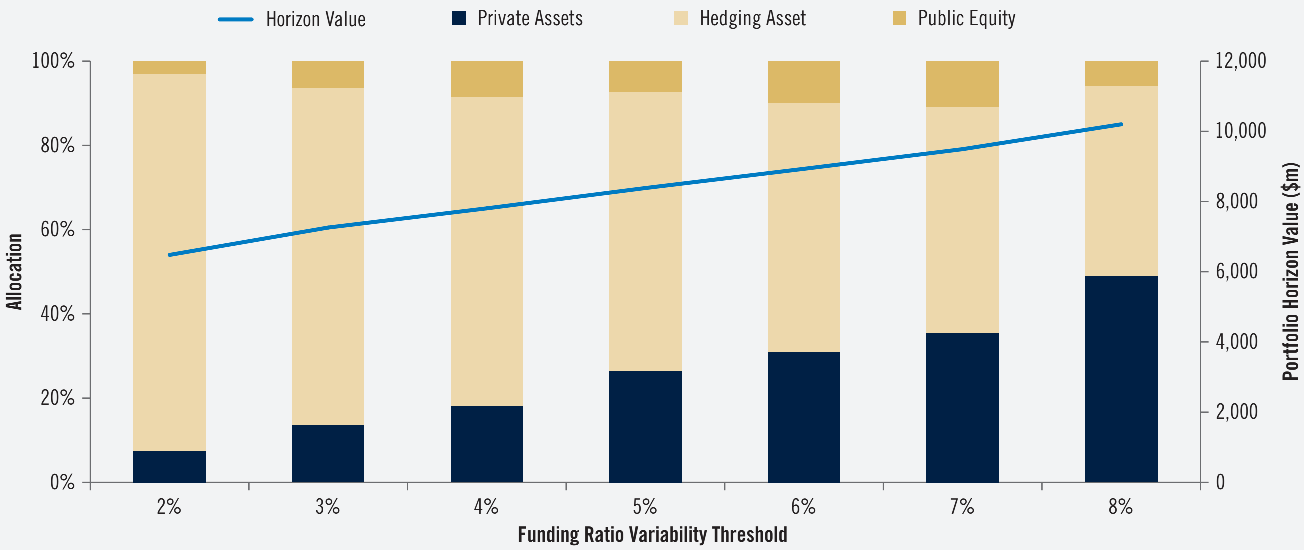
Figure 4 shows the distributions of possible portfolio horizon values for two asset allocations: one for a 2% and the other for a 6% funding ratio variability threshold. As expected, the distribution for the 2% threshold is more concentrated while the distribution for the 6% threshold is more dispersed and positively-skewed. As the funding ratio variability threshold increases (*i.e.,* becomes looser), the allocation to potentially more rewarding private assets increases, resulting in increased expected horizon value and volatility.

Funding Ratio Dynamics

To illustrate funding ratio dynamics of the baseline scenarios Figure 5 shows the funding ratio distribution at 1y, 5y, and 10y horizons. The light blue areas represent distributions of the funding ratios (from all successful simulation runs out of the 5,000 total simulation runs) for the optimal asset allocation constrained on a 2% funding ratio variability threshold while the dark blue areas represent distributions constrained on a 6% threshold. It is observed that, first, at any given time over the investment horizon, a higher (*i.e.,* less restrictive) funding ratio variability threshold results in a more dispersed funding ratio distribution. For example, in year 5 for a plan with an initial funding ratio of 85%, the range of possible funding ratios with a 6% threshold (between ~75% and ~110%) is wider than for a 2% threshold (between ~80% and ~90%). Second, when we focus on one funding ratio variability threshold (*e.g.,* 6%), as time elapses the funding ratio distribution becomes more dispersed. For example, at year 10 the funding ratio distribution covers a much broader range than in year 1.

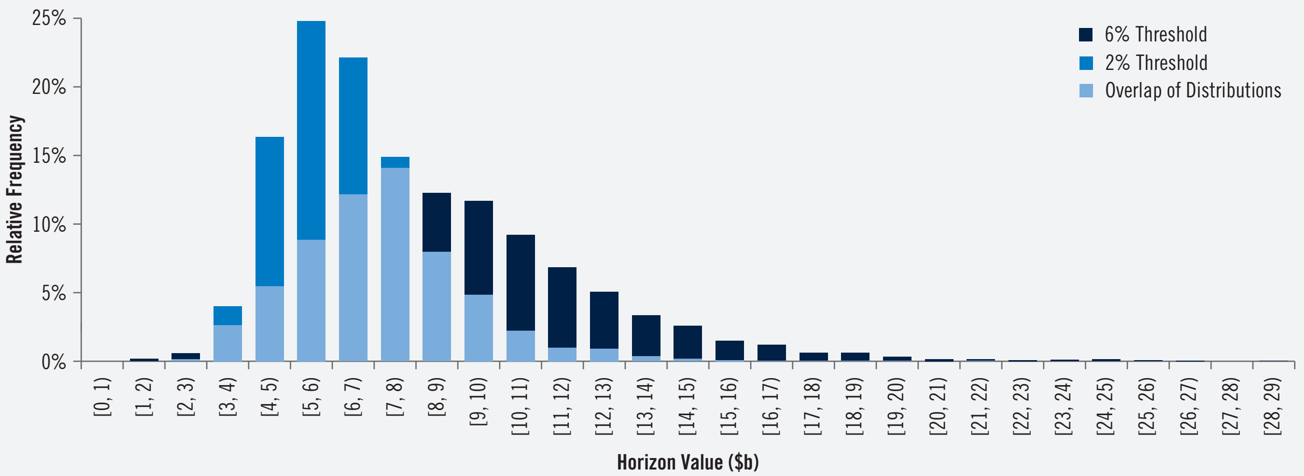
Figure 6 shows the evolution of the range of the funding ratio over time. For any given time during the investment horizon, the 95% confidence interval for the funding ratio with a 6% threshold is always wider than that for the 2% threshold. And if we only look at the range for one funding ratio variability threshold (*e.g.,* the range for a 6% threshold within the dark blue dotted lines), it becomes wider and wider over time.

Figure 3: Tradeoff between Funding Ratio Variability Threshold and Portfolio Performance



Source: PGIM IAS. Provided for illustrative purposes only.

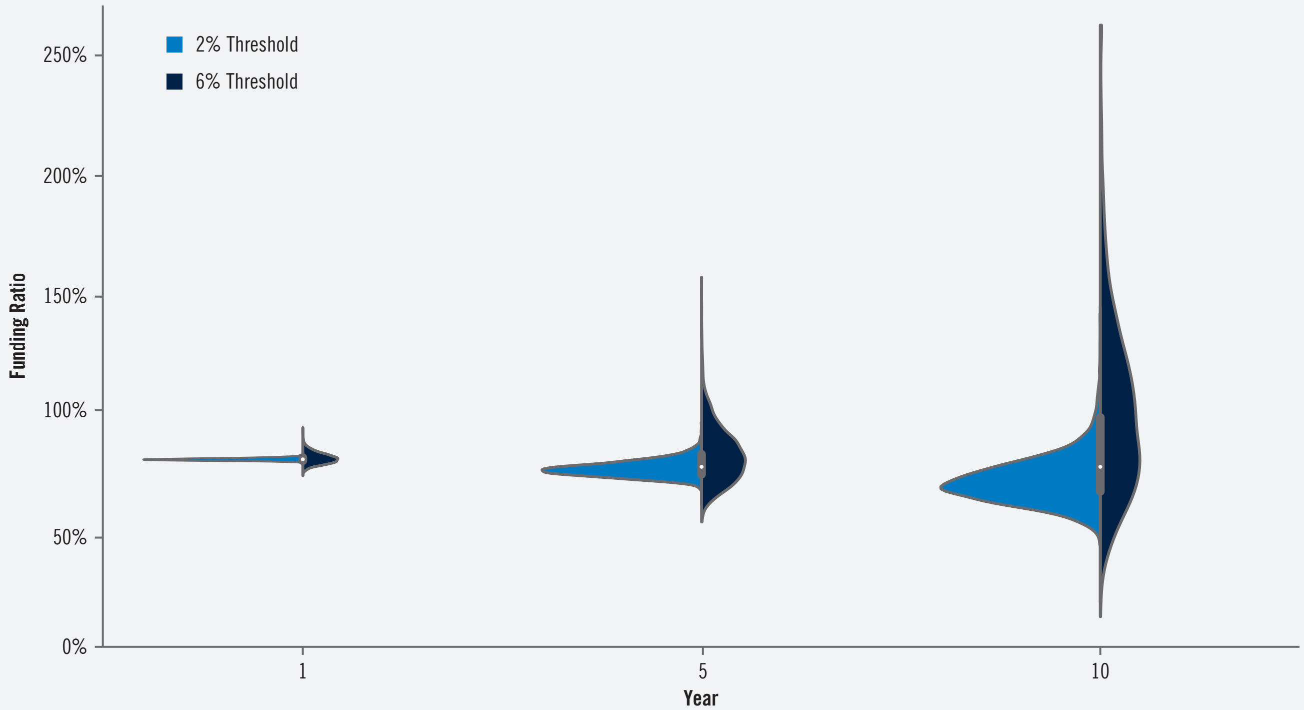
Figure 4: Portfolio Horizon Value Distribution
(Funding Ratio Variability Threshold of 2% and 6%)



Note: This example shows the impact of changing the funding ratio variability threshold on the portfolio's horizon value (10y horizon) with 99% confidence level. The histogram shows horizon values for 6% and 2% funding ratio variability thresholds. The asset allocation used for this example is 59% hedging asset, 10% public equity, 6% LP mezzanine debt, 3% LP real estate and 22% LP buyout for a 6% funding ratio variability threshold; and 89% hedging asset, 3% public equity, 3% LP mezzanine debt, 2% LP real estate and 3% LP buyout for a 2% funding ratio variability threshold. Hypothetical example provided for illustrative purposes only.

Source: PGIM IAS.

Figure 5: Funding Ratio Distribution



Note: The figure shows funding ratio distributions for 6% and 2% funding ratio variability thresholds. The asset allocation used for this example is the same as described in the note to Figure 4. Funding ratio distribution for a 6% funding ratio variability threshold at 10y is winsorized at 0.5%. Hypothetical example provided for illustrative purposes only.

Source: PGIM IAS.

Towards the end of the 10y horizon, some simulation runs (out of 5,000) produce high funding ratios, (*e.g.,* above 150%) for a 6% threshold. There are multiple factors driving this. First, the plan is initially well-funded so scenarios where the assets perform particularly well will produce very high terminal funding ratios. Second, in the simulation runs exhibiting very high 10y horizon funding ratios, the total illiquid assets as a percentage of the overall portfolio value generally keeps increasing over time and reaches a high level. The higher expected performance of the illiquid assets continues to boost overall portfolio returns as the 10y horizon approaches and results in a higher 10y funding ratio.[12]

Asset Allocation Incorporating Mortality Uncertainty

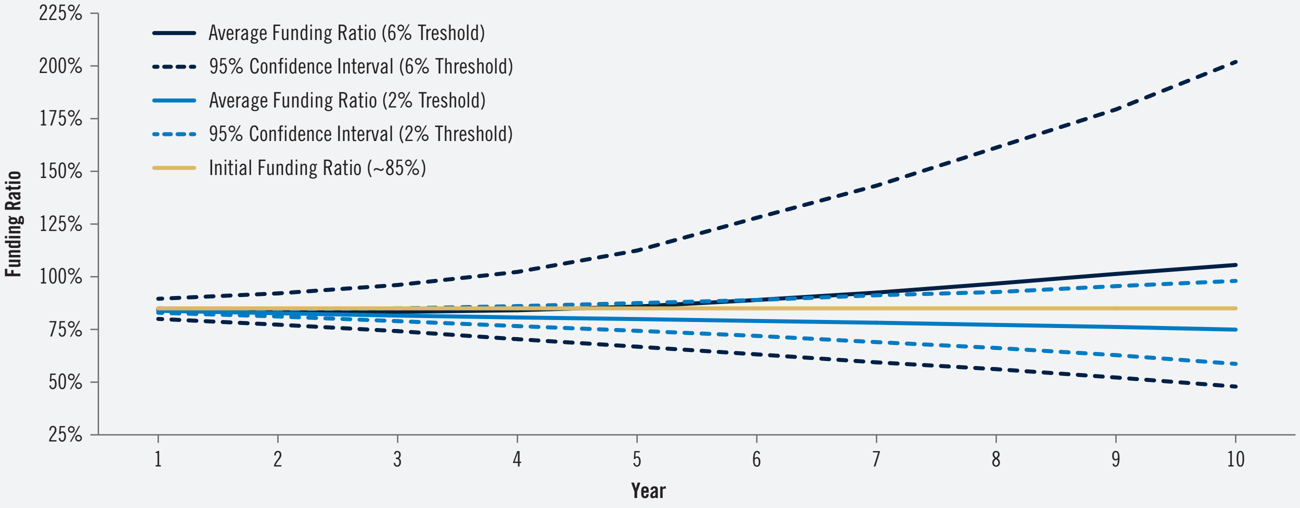
Uncertainty regarding plan participant mortality may cause actual cash liabilities of an end-state portfolio to deviate from the projected liability schedule. Various triggers can make the mortality experience different from the one assumed. For example, a bad flu season may lead to more deaths and make the actual pension liabilities less than projected during those months. On the other hand, improvement in cancer treatment techniques may help people live longer relatively soon (*e.g.,* 5 years from now) that makes future pension liabilities higher than currently expected.

Given the unpredictability of mortality changes, modeling mortality uncertainty and its effect on pension plans' liabilities and optimal asset allocation is challenging. A recent paper explored the impact of longevity risk on pension plans and concluded that the risk is small relative to market risk over a one year period.[13] However, we can improve the evaluation of the impact of mortality uncertainty by dynamically modeling the discount rate over time (*i.e.,* beyond a one year measure) to examine the effect on both liability and asset sides, analyzing the funding ratio dynamics and understanding tail risk better. In this section, we incorporate some mortality uncertainty into the hypothetical end-state portfolio Case Study baseline liability schedule and evaluate its effect on asset allocation.

12 Please see Appendix A6: "Funding Ratio Evolution in Individual Simulation Runs" for a detailed illustration.

13 See "Putting Longevity Risk in its Place," NISA Investment Advisors, L.L.C., April 2013.

Figure 6: Funding Ratio Range



Note: The figure shows funding ratio evolution for 6% and 2% funding ratio variability thresholds. The asset allocation used for this example is the same as in Figure 4. Funding ratio distribution for a 6% funding ratio variability threshold at 10y is winsorized at 0.5%. Hypothetical example provided for illustrative purposes only.

Source: PGIM IAS.

Impact of Mortality Uncertainty on Future Cash Obligations

Figure 7 shows two sample paths of a projected liability schedule after incorporating some mortality uncertainty, in comparison with the baseline liability path. The dark blue line represents the baseline mortality certain liability schedule (from Figure 1).[14] The light blue line (path I) represents a revised liability schedule after incorporating a 10% lower death rate than currently expected (*i.e.*, lower actual mortality or unexpected longevity improvement) and the yellow line (path II) represents a revised liability schedule after incorporating a 10% higher death rate (*i.e.*, higher actual mortality).

While keeping other baseline assumptions unchanged, the present value of liabilities for path I increases to $12,125m (a 3% increase from $11,772m present value of baseline liabilities) and the initial funding ratio falls to 82% (from 85% of baseline funding ratio). For path II, the present value of liabilities decreases to $11,451m (a 2.7% decrease from the present value of baseline liabilities) and the initial funding ratio increases to 87%. Since path I (unexpected longevity improvement) poses more risks to the end-state portfolio, we will focus on optimal asset allocation results for path I.

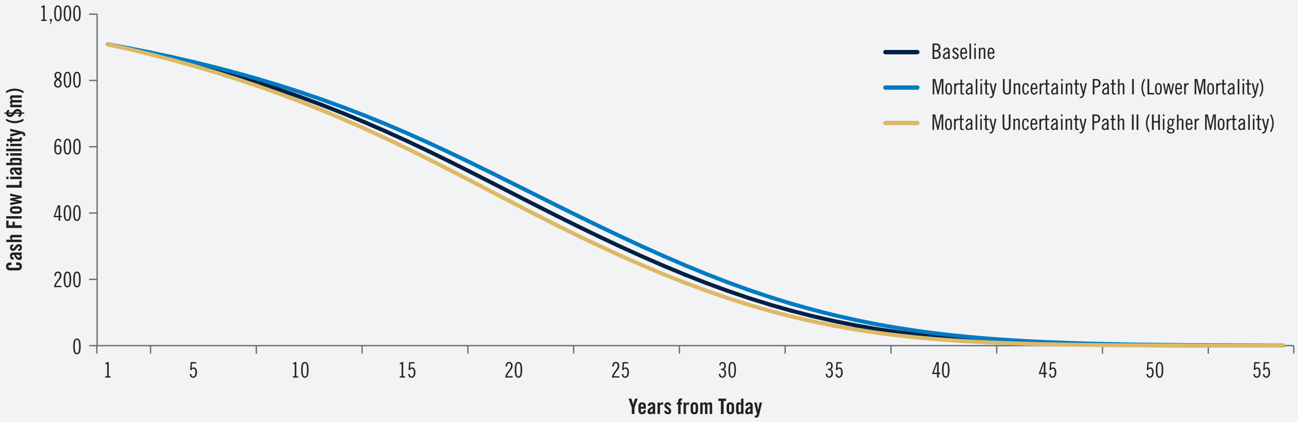
Impact of Mortality Uncertainty on Asset Allocation

Figure 8 shows asset allocation results for sample path I in Figure 7, in comparison with baseline asset allocation subject to the same constraints (99% confidence level; 6% funding ratio variability threshold). Unexpected longevity improvement causes increased allocation to return-seeking assets. The allocation to better expected performing LP buyout funds increases from 22% to 24% and allocation to public equity increases from 10% to 12%. Correspondingly, the expected horizon value increases (from $8,932m to $9,006m) to cover the additional liabilities from the unexpected longevity improvement while also maintaining a desired confidence level.

What if the investor insists on keeping the original asset allocation without adjusting for the unexpected longevity improvement? The expected 10y horizon value of the portfolio falls to $8,858m from the $8,932m in the baseline scenario. Keeping the original portfolio composition derived from the baseline assumptions affects the end-state portfolio's ability to pay the remaining liabilities beyond the 10y investment horizon as those liabilities increase due to the unexpected longevity improvement. In addition, in the baseline scenario the average 10y horizon funding ratio is 106%. However, keeping the baseline asset allocation with 10% unexpected longevity improvement, the average horizon funding ratio falls to 98%. Therefore, ignoring unexpected longevity improvement increases end-state portfolio's funding ratio tail risk. This risk can be mitigated by moving to a new portfolio allocation (see right hand figure of Figure 8) that can improve the average 10y horizon funding ratio to 100%.

14 We add mortality uncertainty to the baseline liability schedule by changing the death rate in the mortality tables while assuming there is no correlation between mortality rate and the economic environment. There are other possible ways to model mortality uncertainty.

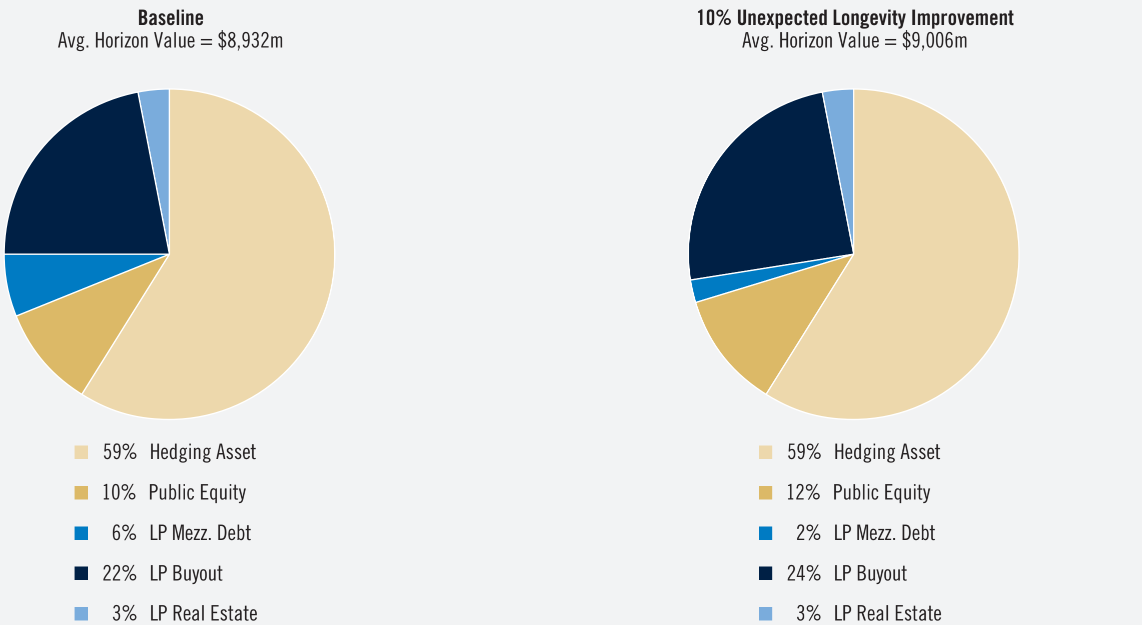
Figure 7: Mortality Uncertainty Impact on End-State Cash Flow Liabilities
(Baseline Liabilities vs. Liabilities with Mortality Uncertainty)



Note: We created the baseline liability profile based on a set of simplified assumptions: a pure retiree population with half 65y old and half 75y old and equal number of males and females in each age group with flat benefit payment for each retiree. We used the RP-2014 mortality table and assumed a flat 1% mortality improvement scale across all ages so that the baseline profile incorporates long-term secular mortality improvement.

Source: PGIM IAS, SOA (Society of Actuaries). Provided for illustrative purposes only.

Figure 8: Asset Allocation Incorporating Mortality Risk
(99% Confidence Level; 6% Funding Ratio Variability Threshold)



Source: PGIM IAS. Provided for illustrative purposes only.

“What-if” Analysis

OASIS is flexible to allow investors to conduct various “what-if” analyses by imposing additional asset allocation constraints or altering certain inputs in the baseline assumptions, and then examining how their end-state portfolio asset allocation changes. In this section, we present examples of three sets of “what-if” analyses, helping investors better understand their portfolios.

Imposing an Upper Limit on Total Allocation to Private Assets

Investment policy may impose limits on a portfolio’s total exposure to private assets. For example, a plan contemplating a future pension risk transfer transaction may wish to constrain the total allocation to illiquid private assets as they are usually not readily accepted by insurers. Therefore, it may be useful to analyze the performance effect of imposing an upper bound on private asset exposure in addition to the other portfolio constraints already discussed. To impose an upper bound on total private exposure during the entire investment horizon, we need to be able to identify the size of the private asset exposure. OASIS estimates the Net Asset Value (NAV) portion of any remaining LP allocation.[15]

We impose an upper limit on total private assets by determining if, in any month, over the 10y horizon the percentage of private asset exposure violates the upper bound limit. If a violation is observed, then 50% of the LP allocation is sold, *pro rata,* and the proceeds (after transaction costs) are reinvested *pro rata* in the public asset portfolio. If a second violation is observed then all of the remaining LP allocation is liquidated and the proceeds, (after transaction costs), are reinvested *pro rata* in the public asset portfolio.[16] Note that without a private asset allocation upper limit, the only trigger of private asset liquidation would be the inadequacy of liquid public assets to fully satisfy a periodic cash flow obligation.

Figure 9 shows the impact of imposing an upper limit of, say 20% on private asset allocations given a funding ratio variability threshold of 6%. As expected, the initial allocation to private assets declines to 11% from 31%. Since OASIS still strives to maximize expected horizon value with the private asset limit constraint, there is a significantly increased allocation to the return-seeking public equity asset from 10% to 27% to maintain performance.

A curious reader may ask: “What if I completely exclude private assets?” In other words, what are the implications of having no private assets at all in the end-state portfolio? Figure 10 shows the asset allocation with no tolerance for private assets in the portfolio. In this case, while the portfolio’s allocation to public equity rises sharply more assets are allocated to the less-volatile hedging asset so as to meet the 6% funding ratio variability threshold. Overall, the expected horizon value falls, decreasing from $8,932m to $8,311m (*i.e.,* a 7% decrease). In this case, eliminating allocation to private assets results in a loss of diversification potential which requires an increased allocation to the hedging asset that contributes to the decrease in the expected horizon value.

Imposing a Floor on the Plan’s Funding Ratio

End-state investors carefully monitor their portfolio’s funded status (as represented by the funding ratio). They may not want the funding ratio of the portfolio to fall below a certain level at any time over the horizon. Therefore, we consider a “what-if” scenario where we impose a floor on the funding ratio. To do so, for each asset allocation, for each simulation run, we determine if the funding ratio at any time during the 10y investment horizon falls below the threshold. An asset allocation where more than 5% of the simulation runs experienced breaches at any time, is no longer considered a potential candidate for the optimal asset allocation solution.

Figure 11 shows that imposing a 55% funding ratio floor limits total allocation to private assets. Now the total allocation to private assets is only 18% *vs.* 31% in the baseline result. The total allocation to private assets decreases noticeably due to their higher volatility. In the meantime, allocation to less volatile hedging asset increases from 59% to 71% to accommodate this additional constraint on the funding ratio floor.

Incorporating Investor Views on LP Performance and Fund-Selection Skills

An important feature of OASIS is that it allows investors to express their views on LP performance relative to public markets and on their fund-selection skills. These views can have a large impact on asset allocation. Here we examine the impact of expressing views on LP performance and fund-selection skills *versus* the baseline of neutral views.

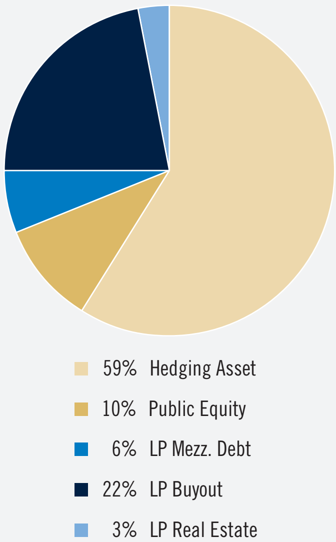
Figure 12 shows the change in asset allocation from expressing positive views on LP buyout performance and above-average LP buyout fund-selection skills. As expected, the allocation to LP buyout increases from 22% to 25%. Also, the relative weight of LP buyout in the overall private portfolio increases from 71% to 76%. The expected horizon value increases from $8,932m to $9,290m.

15 Modeling details of evaluating private asset exposure can be found in Appendix A7: “Modeling Net Asset Exposure for Private Assets.”

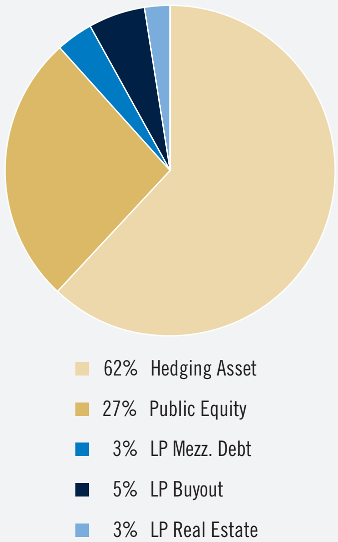
16 See J. Shen, F. Farazmand and Y. Teng “The Tradeoff between Liquidity and Performance: Private Assets in Institutional Portfolios,” PGIM IAS, January 2019, for rationale of this transaction lumpiness.

Figure 9: Upper Limit on Total Allocation to Private Assets
(99% Confidence Level; 6% Funding Ratio Variability Threshold)

Baseline
Avg. Horizon Value = $8,932m



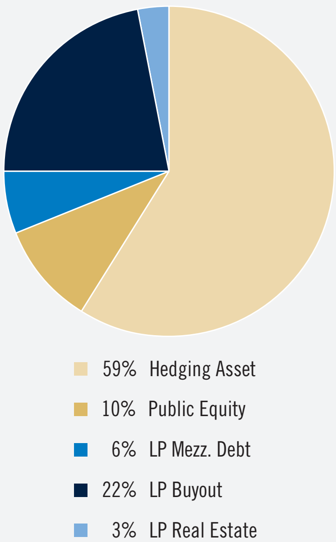
20% Max Allocation to Private Assets
Avg. Horizon Value = $8,621m



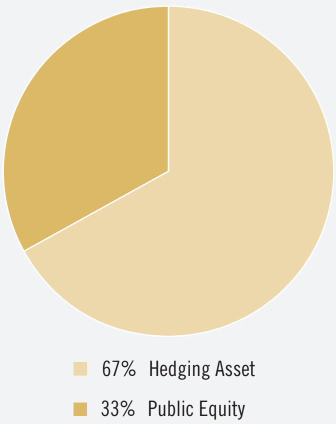
Source: PGIM IAS. Provided for illustrative purposes only.

Figure 10: Public-Only Asset Allocation
(99% Confidence Level; 6% Funding Ratio Variability Threshold)

Baseline
Avg. Horizon Value = $8,932m



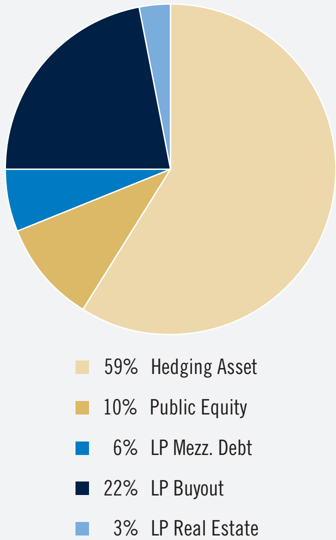
Only Public Assets
Avg. Horizon Value = $8,311m



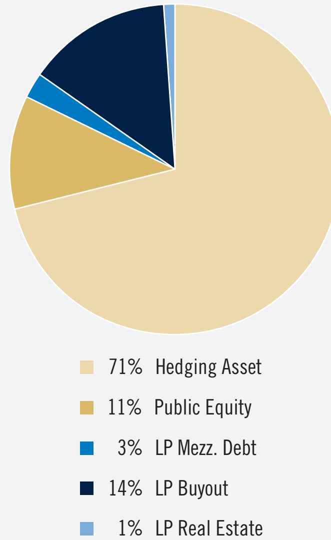
Source: PGIM IAS. Provided for illustrative purposes only.

Figure 11: Imposing a Funding Ratio Floor
(99% Confidence Level; 6% Funding Ratio Variability Threshold)

Baseline
Avg. Horizon Value = $8,932m



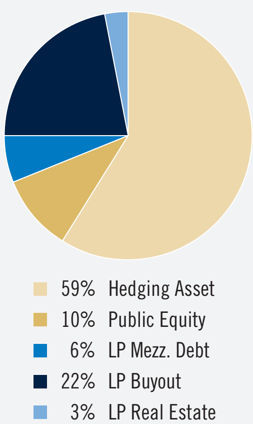
55% Funding Ratio Floor (5% Chance of Violation)
Avg. Horizon Value = $8,055m



Source: PGIM IAS. Provided for illustrative purposes only.

Figure 12: Investor Views and Fund-Selection Skills (on LP Buyout)
(99% Confidence Level; 6% Funding Ratio Variability Threshold)

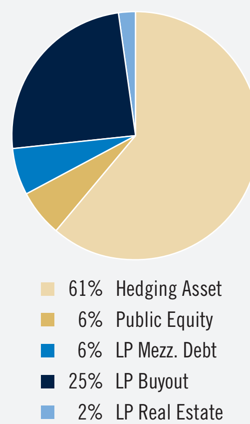
Baseline
Avg. Horizon Value = $8,932m



	Investor Views on Expected Future LP Performance		
LP Type	Buyout	Mezz. Debt	Real Estate
Investor Views	Positive View	No View	No View
Quartile 1 (Q1 - highest)	30%	25%	25%
Q2	40%	25%	25%
Q3	20%	25%	25%
Quartile 4 (Q4 - lowest)	10%	25%	25%

	Investor Fund-Selection Skill		
LP Type	Buyout	Mezz. Debt	Real Estate
Fund-Selection Skill	Above-Average	Average	Average
Quartile 1 (Q1 - highest)	30%	25%	25%
Q2	40%	25%	25%
Q3	20%	25%	25%
Quartile 4 (Q4 - lowest)	10%	25%	25%

Positive Views & Fund-Selection Skills
Avg. Horizon Value = $9,290m



Source: PGIM IAS. Provided for illustrative purposes only.

Summary

Private assets can play an important role in end-state portfolios, given their potential "private market premia" and diversification benefits. We use our simulation-based asset allocation framework, OASIS, that incorporates key characteristics of common public and private assets to help solve for optimal asset allocations – public *vs.* private, as well as the allocation within the public and private portfolios. The public and private portfolios are interrelated, and the overall asset allocation must be jointly determined.

The application of OASIS addresses common concerns expressed by end-state investors – "Maximize expected portfolio horizon value provided I am sufficiently confident of meeting all my future cash obligations over the entire investment horizon, while keeping the funded status of the portfolios sufficiently stable." OASIS also allows investors to conduct extensive "what-if" analyses to address any special requirements such as additional constraints or incorporating certain views.

While the hedging asset helps stabilize funded status, maintaining a reasonable level of allocation to return-seeking private assets can help improve portfolio performance. Even with the most restrictive constraints on both liquidity and funded status stability requirements, there can be a meaningful allocation to private assets. Unexpected longevity improvement can lead to larger-than-expected realized cash liabilities over time, arguing for increased allocation to private assets to cover these additional liabilities. Not surprisingly, less restrictive constraints give more room for allocation to private assets and allocation to high-risk equity assets in the public portion of an end-state portfolio to maximize the horizon portfolio value.

CIOs can use OASIS to evaluate the cost of imposing additional constraints on their end-state portfolios. This analysis can help CIOs make more informed decisions so that their end-state portfolios best fit their overall corporate strategy.

Acknowledgement

We wish to thank Eitan Gazit, Dr. Taimur Hyat, Cameron Lochhead, Tom McCartan, Dr. Sergey Myagchilov and George Silos for their valuable contributions to this project.

Appendix

A1. Summary of Case Study Assumptions

Figure A1: Summary of End-State Portfolio Asset Allocation Case Study Investor Input Assumptions

Investor Inputs			
Current Assets and Liabilities			
Current AUM	$10,000 m		
Liability Schedule	Periodic Cash Liability Schedule		
Public Assets			
	Liquid High-Risk (S&P 500 Index)		
	Return Statistics in "Good" State of Economy		
Annualized Expected Return	13.6%		
Annualized Standard Deviation	12.7%		
	Return Statistics in "Bad" State of Economy		
Annualized Expected Return	4.8%		
Annualized Standard Deviation	18.6%		
Private Assets			
	LP Investment Type		
	LP Buyout	**LP Mezz. Debt**	**LP Real Estate**
Allow? (YES/NO)	YES	YES	YES
Income (%/y)	N/A	6%	4%
	LP Transaction Costs		
"Good" Economy	5%	5%	5%
"Bad" Economy	30%	15%	9%
	LP Capital Call Assumptions		
Stub Period (years)	2	0	1
% Never Called	0%	0%	0%
Default Investment	Public assets portfolio	Public assets portfolio	Public assets portfolio
	Investor Views on Expected Future LP Performance		
Quartile 1 (Q1 – highest)	25%	25%	25%
Q2	25%	25%	25%
Q3	25%	25%	25%
Quartile 4 (Q4 – lowest)	25%	25%	25%
	Investor Fund-Selection Skill		
Quartile 1 (Q1 – highest)	25%	25%	25%
Q2	25%	25%	25%
Q3	25%	25%	25%
Quartile 4 (Q4 – lowest)	25%	25%	25%
	LP Diversification Parameters		
Number of LP Funds	5	5	5
Pension Liability Discount Rate			
Initial Liability Discount Rate	3.9%		

Note: Yellow field indicates an investor input. For public high-risk asset, only the return statistics in "Good" state of economy are user inputs while return statistics in "Bad" state of economy are based on historical data. Instead of specifying a fixed stub period for each private asset, OASIS can also randomize stub periods for each private asset across the 5,000 simulation runs.

Source: PGIM IAS. Provided for illustrative purposes only.

A2. Formulating End-State Portfolio Investment Objective and Constraints

An end-state portfolio investor's portfolio problem is:

Maximize $Expected\ MV\ Portfolio_{Horizon}$

$$\text{s.t., } \Pr\{(MV\ Portfolio_t > Cash\ Liability_t)\ for\ all\ t\ over\ the\ horizon\} \geq Confidence\ Level\ (\%)$$

$$Expected\ Funding\ Ratio\ Variability \leq Threshold\ (\%)$$

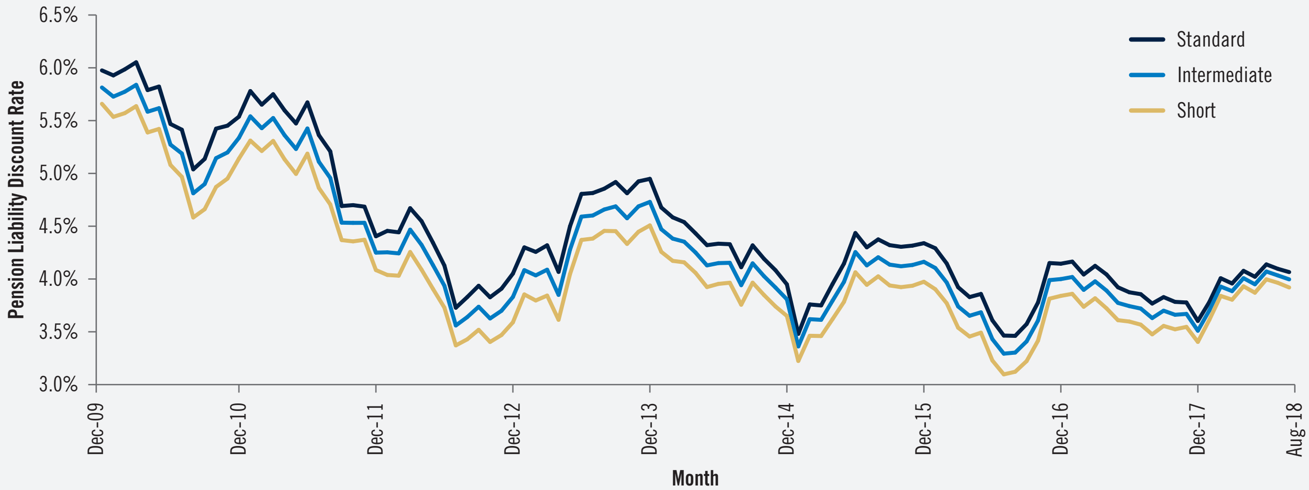
A3. Modeling Pension Liability Discount Rate

The FTSE Pension Liability Index Family

The FTSE Pension Liability Index reflects the discount rate that might be used by corporate pension funds to evaluate the present value of their liabilities. The index is derived from the FTSE Pension Discount Curve (formerly Citi Pension Liability Index and Citi Pension Discount Curve).[17] It represents the single discount rate (a single yield) that would produce the same value as calculated by discounting a standardized set of liabilities using the FTSE Pension Discount Curve. The FTSE Pension Discount Curve represents a set of yields on hypothetical high-quality, AA-rated, zero coupon bonds whose maturities range from 6m to 30y.

In the past decade, as more pension plans partially or fully close, the FTSE Pension Liability Index family evolved. In 2010, two additional sets of discount rates became available to accommodate the needs of pension plans with shorter liabilities. Figure A2 shows the three discount indices in FTSE pension liability index family.

Figure A2: The FTSE Pension Liability Index Family
(December 2009 – August 2018)



Source: FTSE Russell, PGIM IAS. Provided for illustrative purposes only.

Modeling Pension Discount Rates

Given that OASIS is simulation based, to monitor the funding ratio over the entire investment horizon we need to dynamically simulate pension discount rates over the 10y horizon.

To do so, we use the FTSE Pension Liability Index as a starting point. Since the corporate rate is the Treasury rate plus comparable maturity corporate spread, we propose to estimate changes in the pension discount rate (r_t) using simulated public

17 The Citi Pension Liability Index and Citi Pension Discount Curve were acquired in August 2017 by FTSE Russell, a unit of the London Stock Exchange Group ("LSEG") and were renamed the FTSE Pension Liability Index and the FTSE Pension Discount Curve. References to Citi will be replaced with the current FTSE Russell references as they become available. The FTSE Pension Liability Index is often used by plan sponsors to value defined benefit pension plan liabilities in compliance with regulatory requirements specified by SEC and FASB. See https://www.soa.org/sections/retirement/ftse-pension-discount-curve/ for more detail.

market returns, *i.e.*, long maturity (10-20y) Treasury *price returns* (component of total returns due primarily to yield change) and AA-rated long maturity (10-20y) corporate *excess returns net of carry* (component of excess returns due primarily to changes in spread to the Treasury curve):

$$r_t - r_{t-1} = \alpha + \beta_1 \times PR^{Long\ Treasury} + \beta_2 \times ER^{AA\ Corp\ net\ of\ carry} + \varepsilon \quad \text{(A1)}$$

r: Pension Discount Rate

PR: Long Maturity (10-20y) Treasury Price Return

ER: Long Maturity (10-20y) AA-rated Corporate Excess Return (net of carry)

We use regression to estimate the parameters in equation A1. To estimate the AA-rated corporate spread component, we note that the reported historical AA-rated corporate excess return series includes the effect of downgraded AA-rated bonds. Therefore, to exclude the effect of downgraded bonds we use changes in the historical long AA-rated average Treasury spread (OAS) from the index (Statistics Universe). Our estimated AA-rated excess returns will better track changes in pension liability discount rates than if we used the reported index excess returns.

Since we want the AA spread change component of excess return, we remove carry. The estimated AA-rated corporate excess return (with no carry) is:

$$ER^{AA\ Corp\ with\ carry} = \frac{OAS}{12} - \Delta OAS \times Duration \quad \text{(A2)}$$

$$ER^{AA\ Corp\ net\ of\ carry} = -\Delta OAS \times Duration \quad \text{(A3)}$$

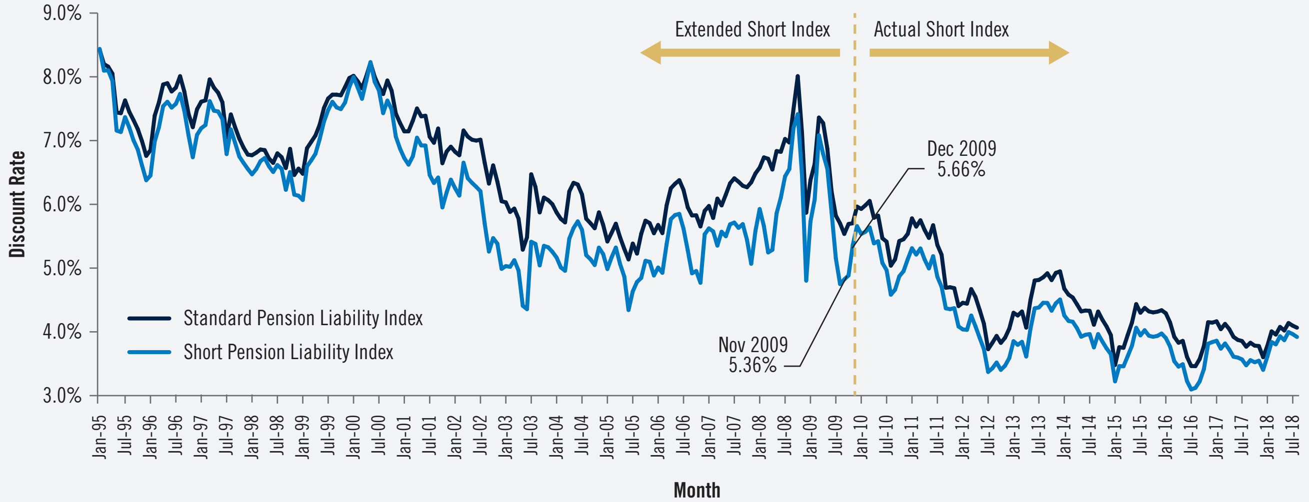
The Short FTSE Pension Liability Index is the relevant index for discounting end-state portfolio liabilities which are usually shorter than those for an open plan. However, the short FTSE Pension Liability Index is available only since December 2009. Therefore, we use the available FTSE Standard Pension Liability Index which is available before December 2009 to construct an extended FTSE Short Pension Liability Index back to 1995. To do so we use the formula below:

$$\textit{Extended Short FTSE discount rate}_t$$

$$= \textit{Standard FTSE discount rate}_t \times (\textit{short AA Corporate Index yield}_t \div \textit{long AA Corporate Index yield}_t) \quad \text{(A4)}$$

This allows us to maintain the spread relationship between the short and long AA-rated corporate bond yields *vs.* short and long FTSE discount rates. Figure A3 shows both Standard and the extended Short FTSE pension liability indices from January 1995 to August 2018.

Figure A3: Pension Liability Discount Rates
(Standard and Short; January 1995 – August 2018)



Source: FTSE Russell, PGIM IAS. Provided for illustrative purposes only.

Figure A4 presents the regression results, and shows that every month, the change of the pension discount rate (from the previous month) is negatively related to both the long Treasury price return and the estimated AA-corporate excess returns.

Figure A4: Pension Discount Rate Estimation

	Pension Discount Rate Change	
	coeff.	t-stat
Constant	0.000	–0.75
$PR^{\text{Long Treasury}}$	–0.093	–21.17
$ER^{\text{AA Corp net of carry}}$	–0.063	–9.97
# observations	283	
R^2	65%	
Data Range	Jan 1995 - Aug 2018; monthly	

Source: Barclays POINT, PGIM IAS. Provided for illustrative purposes only.

For OASIS to dynamically estimate pension discount rates, the investor is required to provide a starting point for the pension discount rate. For example, to be consistent with the current interest rate environment, the starting value could be 3.9% which represents the February 2019 pension discount rate from the Short FTSE Pension Liability Index. Given this initial value, each simulation run uses the estimated regression parameters above to generate a series of future pension discount rates over the entire 10y investment horizon (*i.e.,* a time series of 120 pension discount rates). We conduct 5,000 such simulation runs which in total generate 5,000 time series of 120 monthly pension discount rates based on the simulated public market characteristics in respective months.

A4. Modeling Public Asset Performance

Public assets exhibit different return and risk characteristics under different capital market environments (*i.e.,* "good" *vs.* "bad" state of the economy). To better represent public asset performance dynamics, our public market performance simulation samples from two different sets of public assets statistics, corresponding to good or bad state of the economy, and incorporating path dependency. We define a "bad" state of the economy when the monthly moving average (6m, backward-looking) of S&P 500 cumulative total returns experiences a drawdown of more than −15%.

Figure A5 shows two sets of capital market assumptions for the high-risk public asset in different economic environments (*i.e.,* good and bad state of the economy based on historical observations). OASIS allows investors to specify their capital market expectations in a good state of the economy and the capital market assumptions in a bad state of the economy are based on historical average.[18]

Figure A5: Public Asset Return and Risk Statistics

	Liquid High-Risk (S&P 500 Index)
	Return Statistics in Good Economy (Bad Economy)
Annualized Expected Return	13.6% (4.8%)
Annualized Standard Deviation	12.7% (18.6%)

Note: Historical average based on monthly data from 1995 to 2018.

Source: Barclays POINT, PGIM IAS. Provided for illustrative purposes only.

18 In our Case Study we assume the investor's inputs for capital market assumptions in good state of economy (shown in Figure A1) are based on historical observations. Therefore, they match the numbers shown here in Figure A5.

Instead of sampling public market returns over 120 months (*i.e.,* 10y investment horizon) from the same public asset return and risk statistics (*i.e.,* mean, standard deviation and correlation), every month we first determine the state of the economy. Then, based on the state of the economy in that month we determine which set of capital market assumptions to use to draw samples for that month. Then we sample public market returns for that month (using a set of either good or bad public market statistics).

To represent public market dynamics better, we define a transition matrix to decide which set of capital market assumptions to use before we draw public market returns. We do so because when we are in a good state of the economy, it is unlikely to have a drawdown of the monthly moving average of the S&P 500 cumulative total returns more than −15%, especially if we only draw public asset return samples from the set of statistics in a good state of the economy. In reality, when we are in a good state there is still a chance that the capital market assumptions match those from a bad state and *vice versa.* Therefore, we calculate the transition probabilities based on historical data (same data as above). In any month, if the state of the economy is good, there is a 77% probability to draw from good economy capital market assumptions and a 23% probability to draw from bad economy capital market assumptions; if the state of the economy is bad, there is a 34% probability to draw from good economy capital market assumptions and a 66% probability to continue to draw from bad economy capital market assumptions (see Figure A6).

Figure A6: Economy Transition Matrix
(For Capital Market Assumptions, or CMA)

	Probability to Draw from Good CMA	Probability to Draw from Bad CMA
Good State of Economy	77%	23%
Bad State of Economy	34%	66%

Source: Barclays POINT, PGIM IAS. Provided for illustrative purposes only.

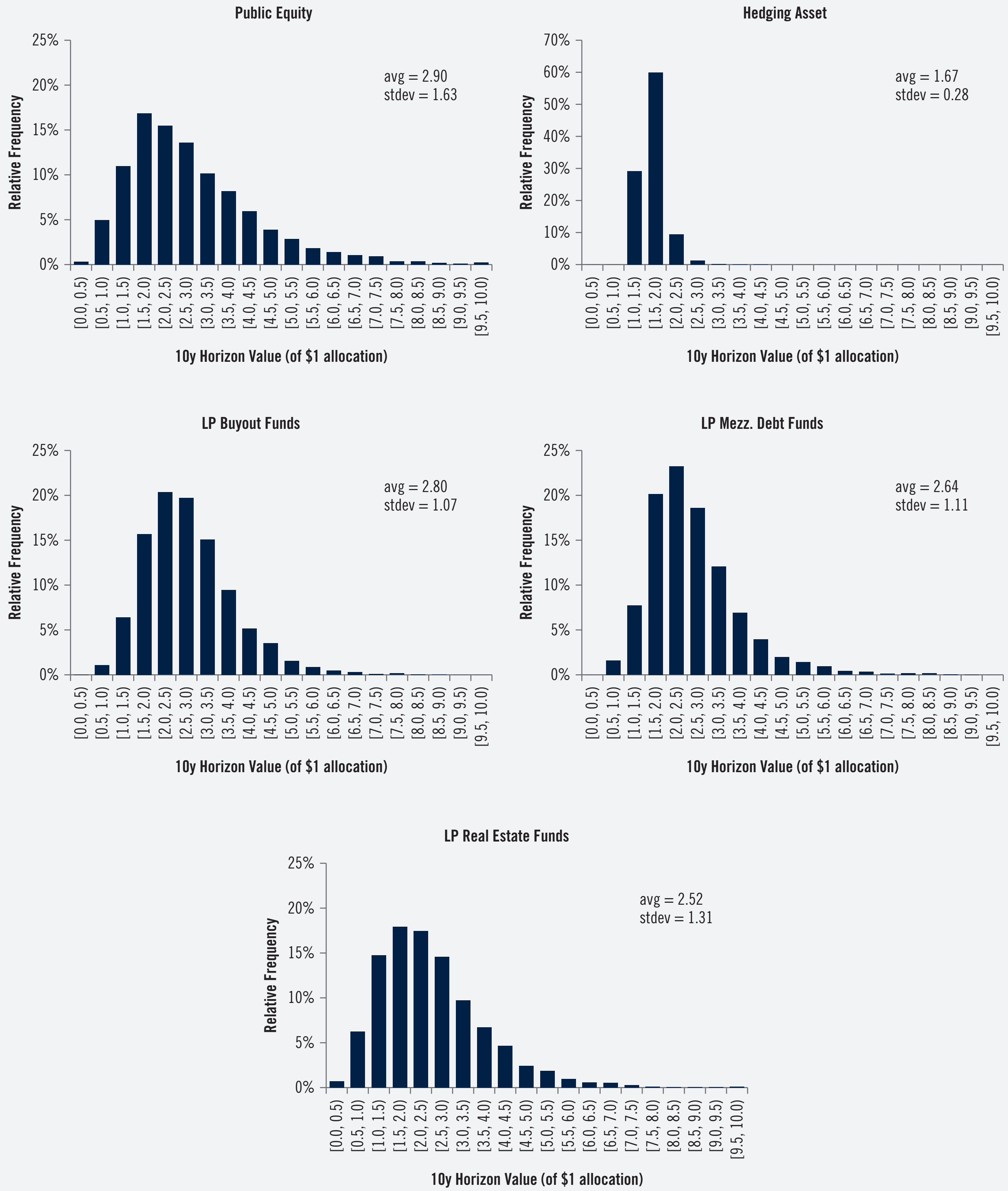
A5. Public and Private Asset Horizon Value Distributions & Correlations

Figure A7 shows the distributions of the 10y horizon values for public assets and the distributions of the LP allocation[19] values for the three private asset types (based on 5,000 simulation runs). These are outcomes based on the Case Study's baseline assumptions on public and private assets.

Figure A8 shows the correlations of the public and private assets 10y horizon values (from simulations). This may help highlight the potential diversification benefits of private assets.

19 OASIS distinguishes between LP allocation and LP investment values by including in the former the horizon value of any undrawn capital. See J. Shen, F. Farazmand and Y. Teng "The Tradeoff between Liquidity and Performance: Private Assets in Institutional Portfolios," PGIM IAS, January 2019.

Figure A7: Public & Private (LP Allocation) Horizon Value Distributions



Source: Barclays POINT, Burgiss, PGIM IAS. Provided for illustrative purposes only.

Figure A8: Correlation of 10y Horizon Values

	Public Equity	Hedging Asset	LP Buyout	LP Mezz. Debt	LP Real Estate
Public Equity	1.00	—	—	—	—
Hedging Asset	(0.11)	1.00	—	—	—
LP Buyout	0.82	(0.01)	1.00	—	—
LP Mezz. Debt	0.81	(0.08)	0.72	1.00	—
LP Real Estate	0.53	(0.03)	0.49	0.46	1.00

Source: Barclays POINT, Burgiss, PGIM IAS. Provided for illustrative purposes only.

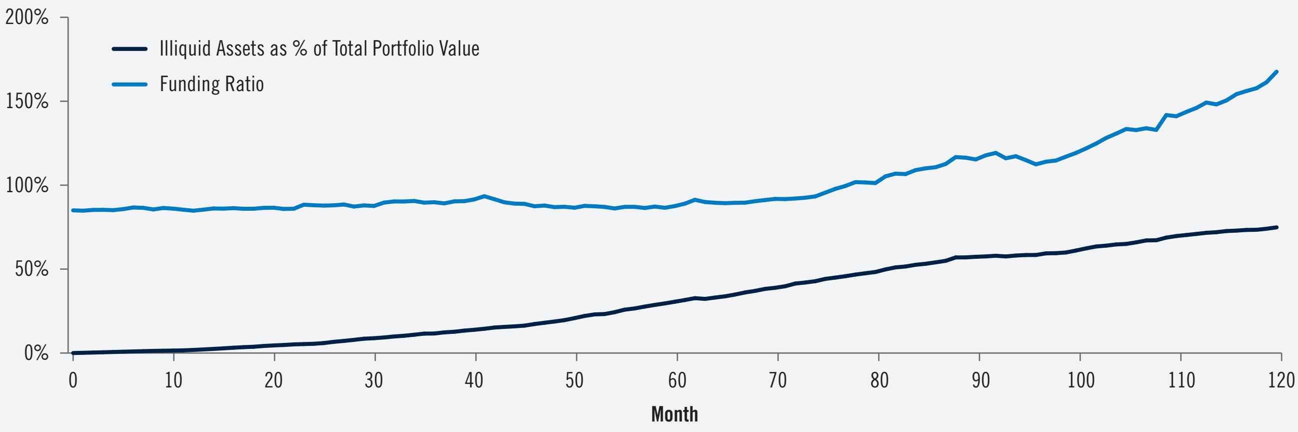
A6. Funding Ratio Evolution in Individual Simulation Runs

Figure A9 shows an example of one simulation run for the baseline portfolio with 6% funding ratio variability threshold that results in a very high (168%) 10y horizon funding ratio. In this simulation run, as time elapses, the percentage of total return-seeking illiquid assets in the overall portfolio continues to rise until it reaches 75% at the 10y horizon. This continues to improve the overall portfolio expected return and consequently the funding ratio, until it reaches 168% at the 10y horizon.

In comparison, Figure A10 shows another simulation run for the same baseline portfolio with 6% funding ratio variability threshold that has a more stable funding ratio throughout the 10y horizon. For this simulation run, as time elapses, the percentage of total illiquid assets in the overall portfolio slowly rises to a much lower level of 32%, allowing the portfolio to maintain a relatively stable funding ratio over time over the 10y horizon.

Figure A11 shows histograms for the 10y horizon percentage of illiquid assets in the total portfolio across all 5,000 simulation runs for both the 2% and 6% thresholds. At the end of the 10y horizon, for a 2% threshold, the percentage of illiquid assets in the portfolio is much lower than that for a 6% threshold (*i.e.,* on average 22% for a 2% threshold *vs.* 68% for a 6% threshold).

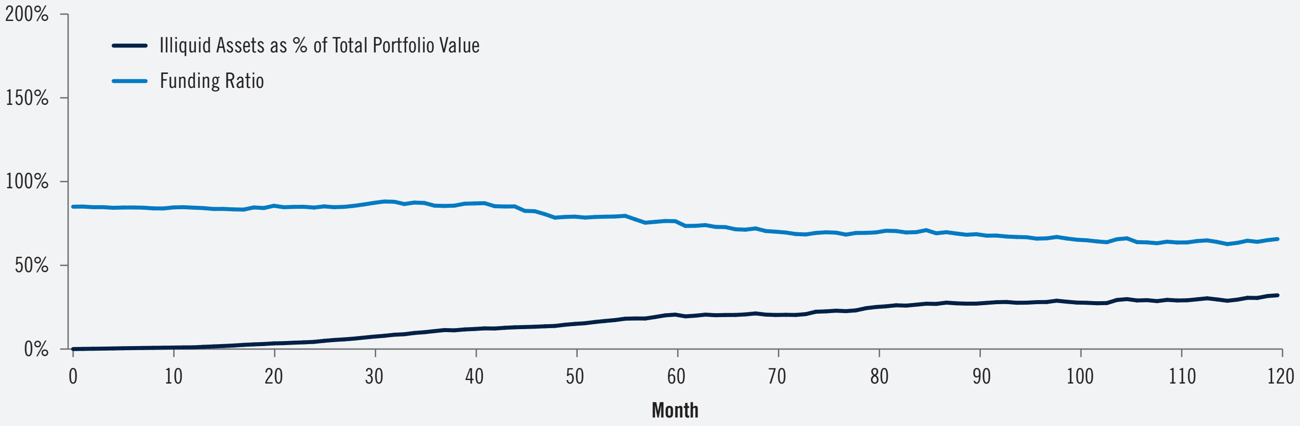
Figure A9: Single Simulation Run with High 10y Funding Ratio
(Funding Ratio and % of Illiquid Asset in Total Portfolio)



Note: The asset allocation used for this example is the optimal asset allocation with 59% hedging asset, 10% public equity, 6% LP mezzanine debt, 22% LP buyout and 3% LP real estate for a 6% funding ratio variability threshold. One simulation run is shown (out of 5,000 runs in total).

Source: PGIM IAS. Hypothetical examples provided for illustrative purposes only.

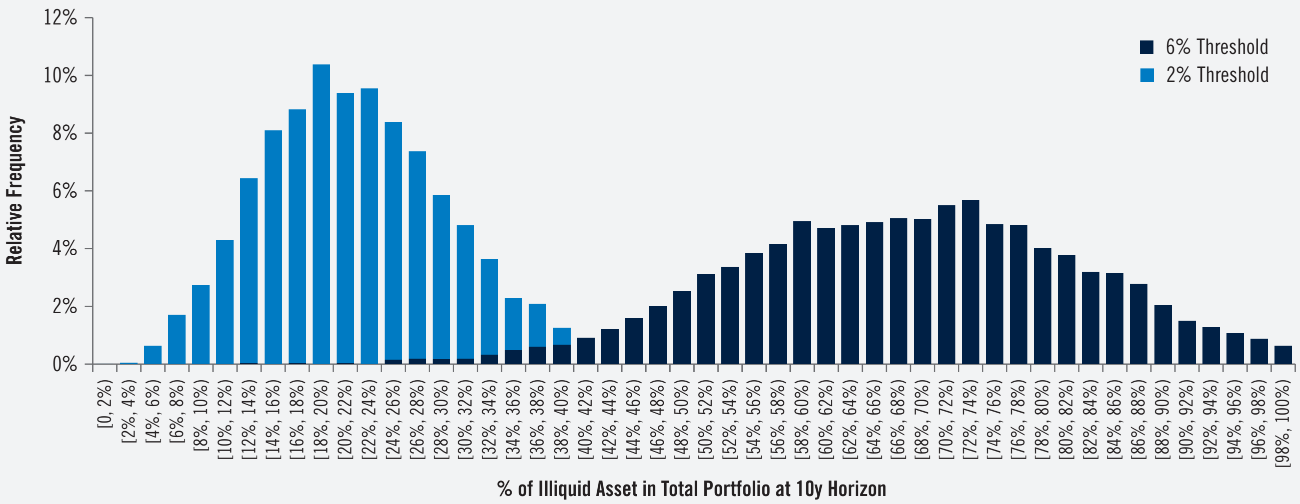
Figure A10: Single Simulation Run with Stable 10y Funding Ratio
(Funding Ratio and % of Illiquid Asset in Total Portfolio)



Note: The asset allocation used for this example is the optimal asset allocation with 59% hedging asset, 10% public equity, 6% LP mezzanine debt, 22% LP buyout and 3% LP real estate for a 6% funding ratio variability threshold. One simulation run is shown (out of 5,000 runs in total).

Source: PGIM IAS. Hypothetical examples provided for illustrative purposes only.

Figure A11: Horizon Percentage of Illiquid Asset in Total Portfolio Distribution
(Baseline Portfolios)



Note: The asset allocation used for this example is 59% hedging asset, 10% public equity, 6% LP mezzanine debt, 3% LP real estate and 22% LP buyout for a 6% funding ratio variability threshold; and 89% hedging asset, 3% public equity, 3% LP mezzanine debt, 2% LP real estate and 3% LP buyout for a 2% funding ratio variability threshold. Hypothetical example provided for illustrative purposes only.

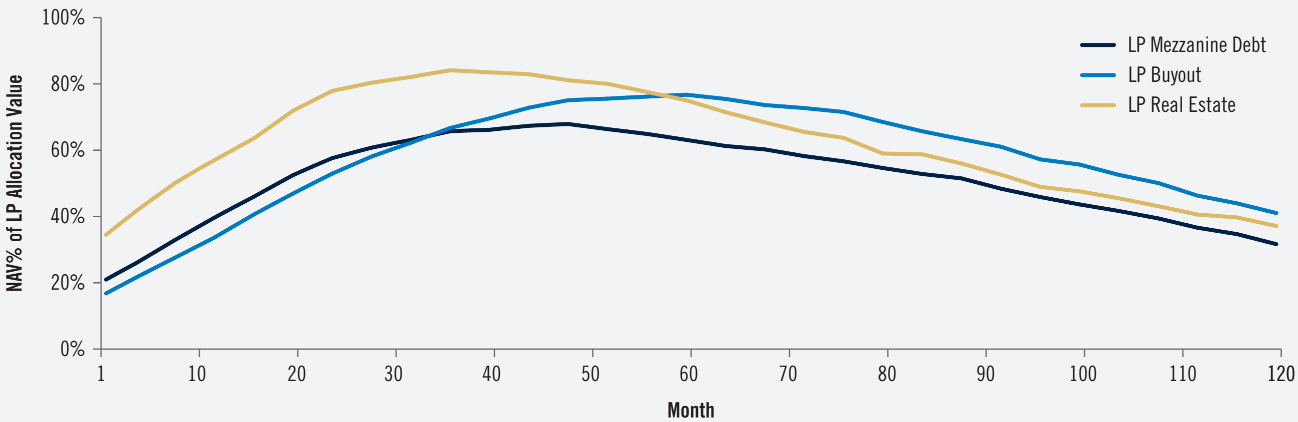
Source: PGIM IAS. Provided for illustrative purposes only.

A7. Modeling Net Asset Exposure for Private Assets

The value generated by an LP investment is the distributions received by the LP and any remaining net asset value (NAV). Unlike public assets which allow for reinvestment, GP distributions are effectively "disinvestments" that reduce the LP's exposure to the private asset. The investment's NAV (as a percentage of the committed capital) represents the prevailing private asset exposure. However, the LP's total investment value consists of both the NAV and the value of the distributions *re-invested* in a public portfolio. As time passes and more distributions are made, less value proportionally is retained in the private asset (NAV) and the LP investment value will increasingly represent the current value of the distributions invested in a public portfolio.

To capture the underlying private asset exposure over time, we estimate the portion of the LP investment value that is solely related to the NAV. The derivation of this fraction is based on observed historical vintage-level LP investment value data. For each month, the estimated NAV ratio (the ratio of NAV to LP investment value) is the average across all vintages. OASIS uses the LP allocation value (LP investment value + future value of uncalled capital) to capture the private asset performance experienced by the LP. Before the GP has completed calling all capital commitments, the LP invests uncalled capital in a public portfolio. Figure A12 shows the approximate NAV as a percentage of the LP allocation value.

Figure A12: Private Asset Exposure as Percentage of LP Allocation Value



Source: Burgiss, PGIM IAS. Hypothetical examples provided for illustrative purposes only.

Important Information

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