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A FAIR COMPARISON FRAMEWORK: RISK AND REWARD IN PRIVATE & PUBLIC INVESTMENTS

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How does an asset with a higher expected return but higher risk compare to an asset with a lower expected return but lower risk? A natural answer is to rank them based on their risk-adjusted returns. But what if the expected return and risk are not estimated reliably? This is the challenge of investing in private markets and comparing their performance with public markets. We offer a framework to fairly compare private and public investment performance.

We present a methodology to reliably estimate the expected return and risk of private assets using the notion of a self-contained, self-financed portfolio. Our estimates are intuitive as they are based on terminal wealth outcomes (rather than a time-series analysis) resulting from investing in private markets. Using these estimates, we compare a variety of investments including PE funds, private debt, public equity and bond indices. Our comparison also accounts for the limits and cost of leverage, when applicable.

We find that terminal wealth-based means and volatilities of private investment returns are significantly different from those computed using traditional time-series return observations. We show that the ranks of various investments based on levered returns (with interest, fees, expenses and manager alpha) can be potentially different from those based on unlevered returns. Importantly, levered returns in mezzanine investments are competitive with buyout investments and that investment in long public market Baa-corporate bonds are, when levered to match risk, competitive with private market investments.

The findings shown are derived from statistical models. Reasonable people may disagree about the appropriate model and assumptions. Models should not be relied upon to make predictions of actual future account performance. See additional disclosures.

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Institutional investors have increasingly allocated to illiquid private assets for their potential diversification and private asset premium benefits. Institutional private asset opportunities are today broadly available and CIOs must decide how to allocate their marginal portfolio dollar not only between equity and credit, but also between public and private vehicles. The first step in this decision is usually an analysis of historical performance.

Performance comparison across various asset classes is best done on a risk-adjusted basis. However, private assets present a challenge in this regard because of their illiquid nature, the absence of market prices and the confidentiality of the related performance data.[1] The inability to compare the risks of private investments and public assets on a consistent basis is a significant complication in asset allocation decisions for multi-asset class portfolios.

CIOs may ask "What has been the return per unit of risk?" While such a risk-adjusted comparison is certainly desirable, it implicitly assumes that less risky investments can be levered up (at no cost) to match the risk of riskier assets. For a "fair" risk-adjusted comparison of returns one must consider the limits and cost of leverage.

To assist CIOs in their asset allocation decisions we introduce a methodology to estimate the risk of private investments based on *long-term* performance. Risk is assessed via ***terminal wealth dispersion*** rather than using the volatility of interim, short-term performance measures (*e.g.,* quarterly IRRs). For many private assets, short-term performance measures are less reliable due to smoothing and, arguably, are less relevant for investors who generally hold the assets to maturity. Since our methodology can also be applied to liquid public investments, we can analyze the performance of public and private investment using the same lens.

The first step in performing a ***fair risk-adjusted comparison*** is to recognize that there may be differences in an investor's ability to time and size the various investments. An investor can often immediately and fully invest in a public asset over the entire investment horizon. However, the timing and size of an investment in private assets is typically at the GP's discretion.[2] The key idea behind our methodology is the notion of a ***self-contained private asset portfolio*** – a composite portfolio that invests any uncommitted or uncalled capital in a public market index such as the S&P 500 Index or Bloomberg-Barclays Aggregate Bond Index.[3] All capital calls are financed by liquidating a portion of the index holdings and all distributions are absorbed back into the index holdings. At no point in time does capital flow in or out of this composite portfolio, thus making it self-contained and self-financed.

The notion of a composite portfolio for private assets has been around since the introduction of public market equivalent (PME) which is a popular measure of private equity performance.[4] We extend this notion to a self-contained composite portfolio and measure the performance at the portfolio level including both uncommitted and committed, but uncalled, capital – not just the capital that was committed and called.[5] For many liquid public market investments, the composite portfolio is simply the investment itself as the investor is immediately and fully invested over the entire investment horizon.

For private investments (*e.g.,* private equity buyout funds) there is no investable index. Instead, the investor must initially commit to a portfolio of individual funds (say, five funds) and then reinvest any future distributions in a new set of funds. Fund-level data include the GP's "alpha" or more properly, expertise to select, acquire, actively manage and exit the fund's portfolio companies. Given the wide performance dispersion across private funds, a fair performance comparison must also recognize the terminal wealth variability associated with the set of funds initially, and subsequently, selected.

For many public investments there are investable index alternatives (*e.g.,* high yield index, leverage loan index, etc.) that represent the performance of the entire asset class (using market capitalization weights). In addition, there is good data availability. To model liquid public investments, we start with an index return and add a variable manager alpha, less fees. The investor can reinvest any distributions received back into the index. Generating the dispersion of terminal wealth is straightforward.

Estimating the risk of illiquid private market investments has received much attention from both academia and industry. However, most of this work is based on estimating the time-series volatility of short-term performance measures like point-to-point IRRs or modified-Dietz returns that rely on periodic NAV values.[6] Modeling the risk of private equity based on periodic returns has appeal because it fits well with public market conventions. However, the periodic returns of private investments, in addition to being low-frequency and noisy, have econometric issues including serial correlation and smoothing. There is substantial amount of literature on de-smoothing private equity return time series and factor modeling including Bayesian inference.[7]

1 The challenges of risk estimation for private capital are well known and have been documented (see Ljungqvist and Richardson, 2003). The private nature of the market leads to problems such as incomplete datasets and limited access to high-quality data. The illiquid nature of private equity leads to econometric problems such as imprecise, smoothed and low-frequency valuations that further lead to noisy and autocorrelated period returns.

2 The distinction between public and private is not absolute. Some public asset investments (*e.g.*, distressed credit funds) may share timing and size characteristics of private equity funds, while some private investments (e.g., hedge funds) may share timing and size characteristics of public bond and stock funds.

3 See O'Shea (2017) and Shen, et al. (2019) for more details.

4 Kaplan and Schoar (2005).

5 There are two sources of uncommitted capital: 1) A portion of the initial capital that is deliberately kept uncommitted by the LP to pace commitments over several vintages for temporal diversification, and 2) Distributions received from GPs which are held as uncommitted capital until committed to future funds.

6 The modified-Dietz return formula provides a first order approximation for short-term IRRs.

7 See Getmansky, et al. (2004), O'Shea and Jeet (2017) and Lizieri, et al. (2012)

Another body of literature on estimating the risk of private investments falls under the broad heading of "risk in the long run" and is based on the dispersion of terminal wealth.[8] The idea of terminal wealth standard deviation (TWSD) to estimate risk was first suggested by Radcliffe (1994) as an alternative risk measure for mutual funds. For private equity returns, this model relies and measures the volatility of private equity returns using the cross-section of terminal wealth. Long (1999) provides a method to estimate the volatility of periodic returns from the spread (range) of terminal wealth outcomes.

To compute the risk of private investments we invert the observed dispersion of terminal wealth to compute what the volatility of periodic returns must have been, assuming returns are Normal and independent and identically distributed (i.i.d.).[9] Using our framework, we compare several different investments in private and public markets on a risk-adjusted basis. We then compare their performance when leverage is employed to bring their risk to the level of buyout fund investments. We also account for the cost of leverage (e.g., spread over LIBOR and haircut).

Description of Data

To compare the performance of equity and credit in public and private vehicles we rely on several datasets. To measure the private investor's ability to time and size their investments we need detailed cash-flow data by fund. Although we do not have access to such data we can generate **synthetic cash-flow** data using funds' since-inception IRRs and TVPIs which are available.[10] To generate these cash-flow data we use a technique by Cornel (2017) wherein IRR and TVPI together produce a measure of duration (d) that measures the time that a private investment is actually invested.[11] We assume a single contribution (representing all contributions) of a dollar is made sometime in the beginning of each fund's vintage year and a single distribution (representing all distributions) equal to the fund's TVPI is received d years later.[12] We generate synthetic cash-flow data for two subclasses of funds in the U.S. private market: buyout and mezzanine funds.

Our datasets are:

LP Buyout and Mezzanine Funds: Quarterly fund-level since-inception IRR and TVPI data and pooled modified-Dietz return data (all U.S. funds) are from Burgiss. These are net of fees. Manager alpha is included in reported fund returns. There are about 1,100 U.S. buyout and 150 mezzanine funds. Buyout funds contain equity investments in mature companies that result in a change of control. These are typically large transactions that use leverage. Mezzanine funds contain loans that are subordinate to other debt in a firm's capital structure and are backed by little to no collateral. Loans are generally term loans (fixed- or floating-rate) and may contain warrants/conversion rights.

To compare private with public market performance we use the S&P 500 index for the large-capitalization U.S. equity market as well as the following U.S. credit market indices:[13]

10y+ IG Corporates (FltR): These data are from Bloomberg-Barclays and is a broad coverage fixed-rate investment-grade (at the beginning of each month) bond index. We convert periodic index returns to floating-rate using reported index excess returns + 3m LIBOR. We use this index to represent an investment in floating rate, long spread-duration IG corporate bonds. Index returns are gross of fees with no additional leverage.

10y+ Baa x-Financial Corporates (FltR): These data are from Bloomberg-Barclays and is a broad coverage fixed-rate index of long duration, Baa-rated corporate bonds (excluding bonds from financial companies). This index is a subset of the 10y+ IG Corporate index above. We convert periodic index returns to floating-rate using reported index Excess Returns + 3m LIBOR.

10y+ Baa x-Financial Corporates (TR): These are the total return data for the fixed-rate index above. We include this index to compare with its floating-rate version to gauge the (Treasury) duration component to returns over the 2005 – 2018 period.

8 See Kaserer and Achleitner (2005), Weidig and Mathonet (2004), Weidig (2002) and Weidig, et al. (2005).

9 The estimated volatility is also adjusted using the square root of time rule, see Jón and Jean-Pierre Z (2006).

10 Total Value to Paid-In (TVPI) is a popular measure of private investment performance; also known as the investment multiple.

11 More intuitively, the notion of duration for private investments is a time distance between the center of gravities of distributions and contributions. In the absence of access to cash-flow data the duration can be computed using IRR and TVPI. See Jeet (2017) which gives a formula $d = \frac{\log_e(\text{TVPI})}{\text{IRR}}$. Duration is also defined using Direct Alpha (DA) and PME: $d = \frac{\log_e(\text{PME})}{\text{DA}}$, see Gredil, et al. (2014).

12 The use of synthetic cash flows may result in risk underestimation as it reduces the uncertainty in cash-flows timings. To compensate we use Long's (1999) method to obtain an upper bound on this possibly underestimated risk. The simultaneous presence of two competing effects results in a max-min estimate for risk. Actual private capital cash flows (to sample random private capital portfolios) can better estimate risk.

13 Since middle-market direct lending LP funds have recently become popular credit investments, we did consider including Cliffwater's direct lending loan-level index. This index represents an investment in a diversified pool of floating-rate middle-market loans held in the portfolios of business development corporations. Returns are measured at the loan level (not a LP fund) and are gross of fees and any additional leverage. The Cliffwater time series includes both senior and subordinate loans which may make it not fully representative of middle-market direct lending funds which are typically just senior loans. The index is non-investable, but it has broad coverage. The difficulty of including the index in our analysis is that it is not a fixed-pool of private loans. Consequently, there is no sure way to compute the terminal wealth of the pool since at the investment horizon the pool contains both newly-issued (whose valuations are less certain) and maturing loans.

Leverage Loans (B/BB-rated): These data are from S&P and is a broad coverage floating-rate index of broadly syndicated bank loans rated B or BB at the beginning of each month. We use this index to represent an investment in leverage loans. Index returns are gross of fees with no additional leverage.

All public indices are fully investable, but their returns exclude any manager alpha from active management skill and manager fees. To enable a fair comparison with private markets, whose returns reflect the GP's active skill, we add a variable alpha and fixed fees to public index returns. Consequently, public market returns equal index returns plus a manager alpha less management fees.

Naïve Performance Comparison

Figure 1 shows the growth of a dollar in various investment opportunities in the U.S. markets between 2005 and 2018. The observed dip during 2018 Q4 is consistent with the 20% drop in the S&P 500 equity market index. It appears that buyout was the best investment opportunity – by a wide margin.

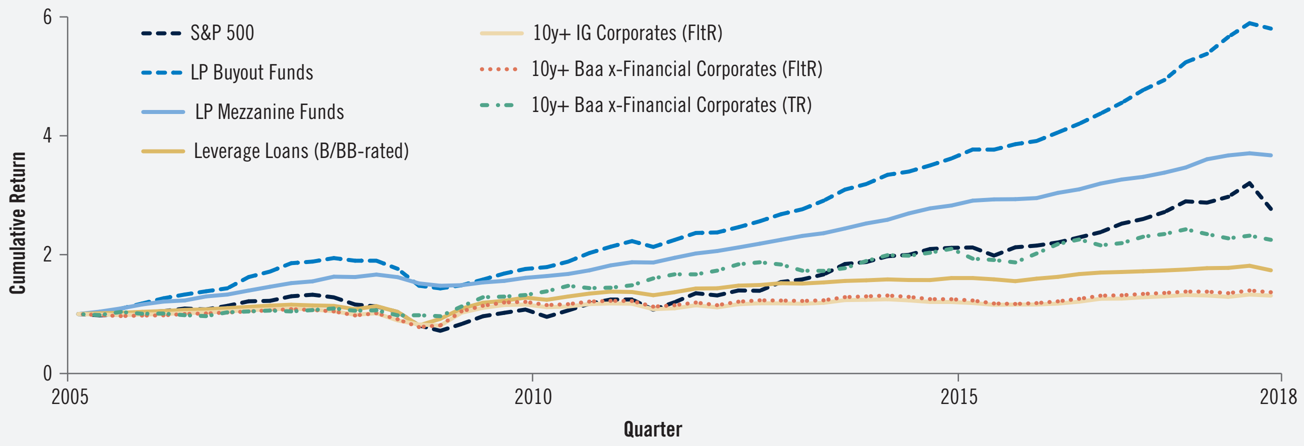
However, the strong buyout (and mezzanine) fund performance in Figure 1 is misleading on several fronts:

1) Replicability: An investor could not have replicated the linked-quarterly private asset returns due to the sheer size of the private market as it would require building a cap-weighted portfolio of every private LP fund, requiring an inordinate amount of capital (given minimum commitment sizes), resources and access. Realistically, an investor would have invested in a small subset of funds, introducing idiosyncratic (fund-selection) risk with the potential for a wider range of performance outcomes over the data period.

2) Self-contained, self-financed: Quarterly private asset returns, computed using either modified-Dietz returns or point-to-point IRRs, are not suitable for linking because these quarterly returns are not from a buy-and-hold (or a self-financed) kind of investment. To replicate these returns one must be able to sustain large cash flows, especially large capital calls. In contrast, the returns of public market investments are self-contained and self-financed.

3) Risk-adjustment: Importantly, the comparison in Figure 1 is not adjusted for risk. As we will show, compared to private markets, public investments have generally exhibited *less volatility*. An investor could have used leverage to try to match the volatility of private investments.

Figure 1: Cumulative Total Returns for Various Investment Strategies
(using linked quarterly returns; January 2005 – December 2018)



Note: For private investments quarterly returns are point-to-point IRRs using cash flows and valuations pooled across all funds in U.S. market for each subclass of funds (buyout and mezzanine).

Source: PGIM IAS, Burgiss, Bloomberg, S&P and Barclays. Provided for illustrative purposes only.

Self-contained Private Investment Portfolio

We can represent an investor's public market portfolio as a *buy and hold* strategy – capital is immediately allocated at the beginning, capital remains fully invested as cash flows are reinvested, and capital is returned promptly at the end of the investment horizon. In contrast, to represent an investor's private market portfolio we must reflect that the investor is continuously managing cash flows whose size and timing are at the GP's discretion. The LP investor will generally commit a portion of their assets to the GP. However, the investor must wait for the commitment to be called and must invest the uncalled, as well as any uncommitted, assets elsewhere.

Can we replicate a private market investment as a buy and hold strategy, at least in theory? We consider the idea of a **self-contained private asset portfolio** which is a composite portfolio wherein any committed, but uncalled and uncommitted capital is invested in a liquid public index (a "default investment") such as the S&P 500 index. All capital calls are financed from, and all distributions are absorbed into, this default index investment.[14]

For a fair performance comparison, we cannot allow distributions to remain in the public default investment. If we did, then the private investment will be mostly a public investment towards the latter years of the investment horizon. Consequently, we also need to specify a rule by which any uncommitted assets are committed to new private LP funds. For our analysis, we assume that half of any uncommitted assets are allocated from the public default investment to new funds in the next period. (Our framework can accommodate more advanced commitment strategies.) New commitments are made periodically (say, per vintage year) based on the uncommitted assets that grow through distributions received. At no point does any of the investor's assets flow in or out of the composite portfolio. In practice, a private investor expends a lot of effort to manage cash flows (buy/sell the default investment), build GP relationships, select funds, make commitments, etc. Our method simplifies the process and assumes a self-contained private asset portfolio in which total assets always stay invested either in the private investment or in the default index investment.[14]

The advantage of a self-contained composite portfolio is that it is easy to measure its performance because at any point you can observe the total value of the portfolio (*i.e.,* wealth). Note that this is different from popular private investment performance measures such as IRR, TVPI, PME, or Direct Alpha that measure performance only on the capital that has been called, distributed, or is in the form of valuation. Our measure of total wealth also considers the assets that are uncalled or uncommitted. It is much simpler to measure total wealth, as it does not require any formula (TVPI, modified-Dietz, or PME) or a root-finding mechanism (IRR or Direct Alpha). Total wealth is truly an accountant's view of performance and the question of time- or money-weighting does not arise. Over a sufficiently long period (say, 10 years) and with a commitment strategy that diversifies private investments (both cross-sectionally and over time), the influence of subjectivity of interim valuations on performance can be minimized as some errors in NAVs may cancel out others.

Risk Estimation – Terminal Wealth Dispersion

Terminal wealth dispersion, rather than the variability of periodic returns over the investment horizon, is a way to capture the risk of private investments. Since an LP investor knows their capital is unavailable until returned by the GP, the investor is concerned about the range of possible asset valuations at the horizon. While terminal wealth dispersion may be considered a superior measure of risk in general, its use is largely absent in public markets since they can be entered and exited at any time at transactable valuations. Importantly, the volatility of periodic returns and the dispersion of terminal wealth are linked. More volatile interim returns are directly associated with more terminal wealth dispersion. Consequently, we can invert the distribution of terminal wealth to estimate the mean and volatility of returns. The crucial assumption for this inversion is that the periodic returns are independently and identically distributed (i.i.d.), which is a milder assumption than used in IRR computations that assumes a constant rate of return. For tractability we further assume that periodic returns are Normally distributed. Figure 2 shows a hypothetical example of large dispersion in terminal wealth arising from a 15-year period of i.i.d. Normal returns.

We use Monte Carlo simulation to generate a distribution of terminal wealth for a self-contained private investment portfolio that follows a given LP fund commitment strategy. Figure 3 shows the dispersion of terminal wealth for buyout and mezzanine portfolios following our simulation procedure. From this distribution we compute the volatility of periodic returns.[15] To do so we proceed as follows:

Let μ and σ be the mean and standard deviation of unobservable but i.i.d. period returns and let W_{max} and W_{min} be the largest and smallest terminal wealth, respectively, observed from the simulation. For a commitment strategy that is run for N vintage years, the estimates of μ and σ can be computed from the following equations:[16]

14 We assume that a self-contained, self-financed portfolio is long-only. In practice, such a portfolio could run into a liquidity issues such as not having enough liquid capital to service capital calls. However, when using synthetic cash flows this does not happen because all the commitments are called immediately. In our simulated portfolios any loss of capital invested in either index or private investments can only reduce the size of future commitments.

15 See Long (1999).

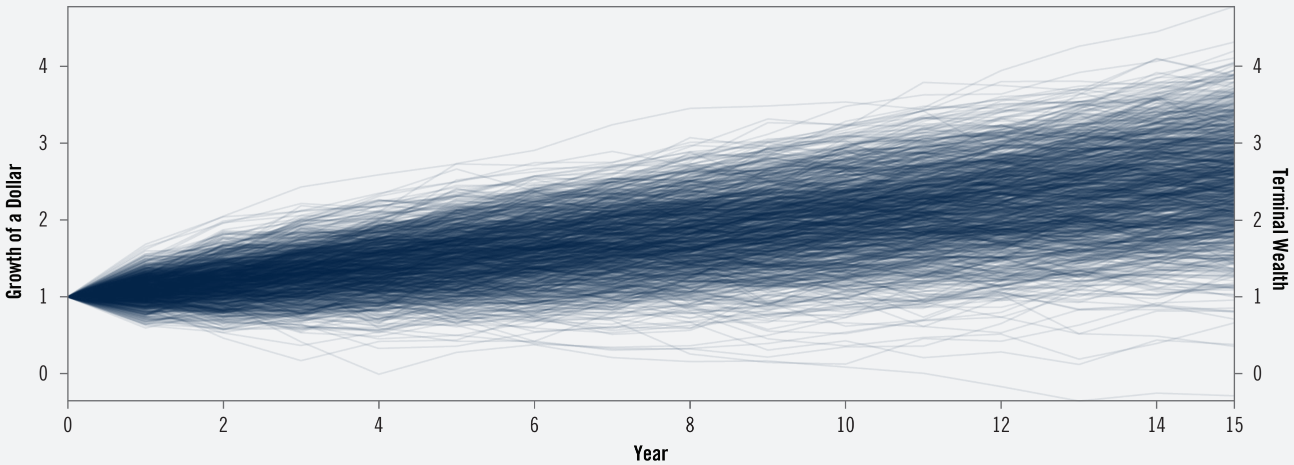
16 Ibid.

$$\mu = \frac{1}{2N}\log(W_{max}\times W_{min}), \text{ and}$$

$$\sigma = \frac{1}{2\sqrt{N}}\log\left(\frac{W_{max}}{W_{min}}\right).$$

We assume the private portfolios follow a very simple commitment strategy in which half of any uncommitted capital at the beginning of each vintage year is committed, equally divided, to *five* randomly sampled funds from the vintage. Starting with a dollar in the beginning of vintage year 2005 we follow the strategy for 14 years until 2018 when we take account of the portfolio's total wealth including any valuations.

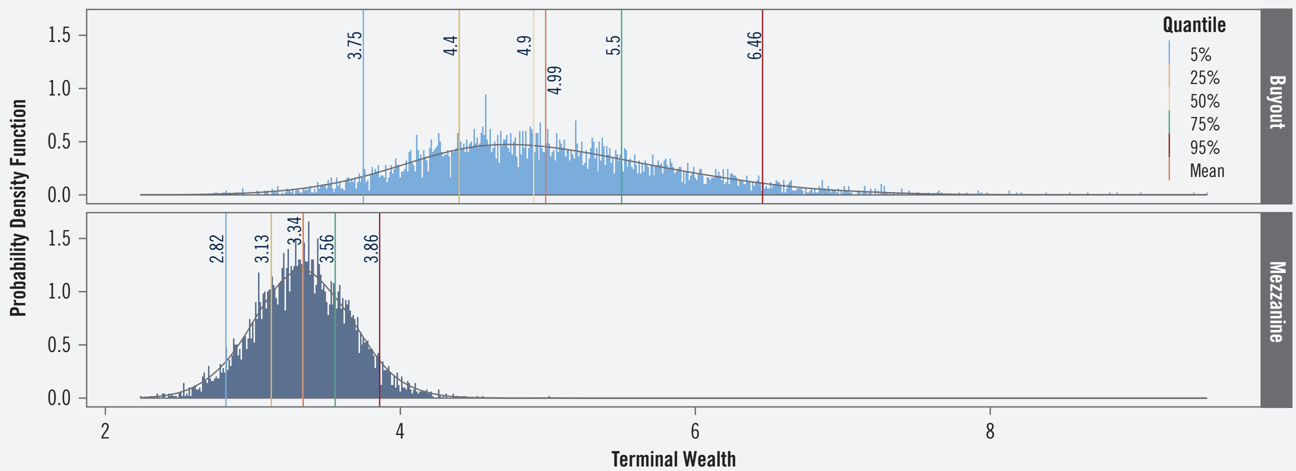
Figure 2: Periodic Returns and Terminal Wealth Dispersion at Year 15 – Hypothetical



Note: Periodic returns are assumed to be i.i.d. Normal. Linking these returns can result in very different terminal wealth.

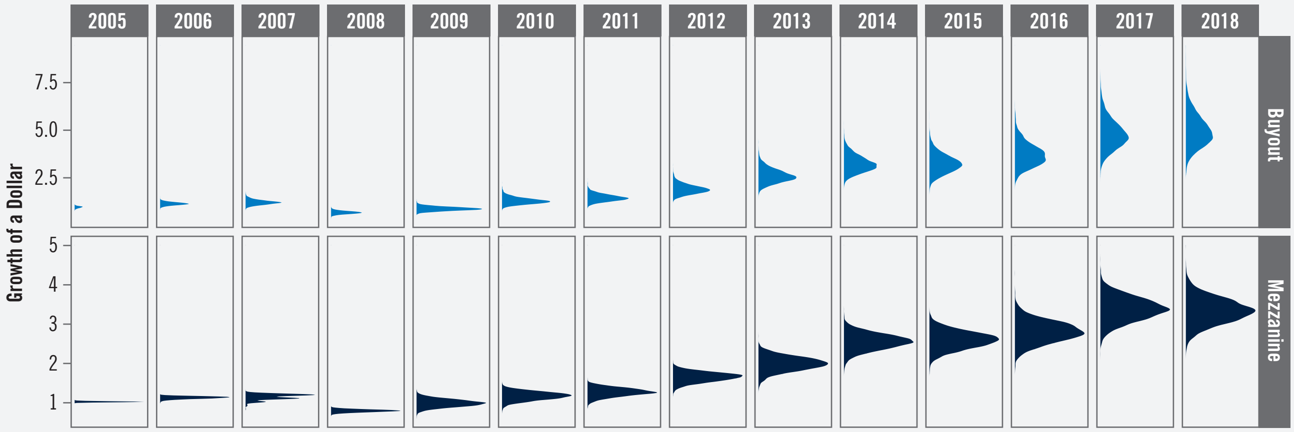
Source: PGIM IAS. Provided for illustrative purposes only.

Figure 3: Terminal Wealth Dispersion of Self-contained Private Investment Strategies



Source: PGIM IAS, Burgiss and S&P. Provided for illustrative purposes only.

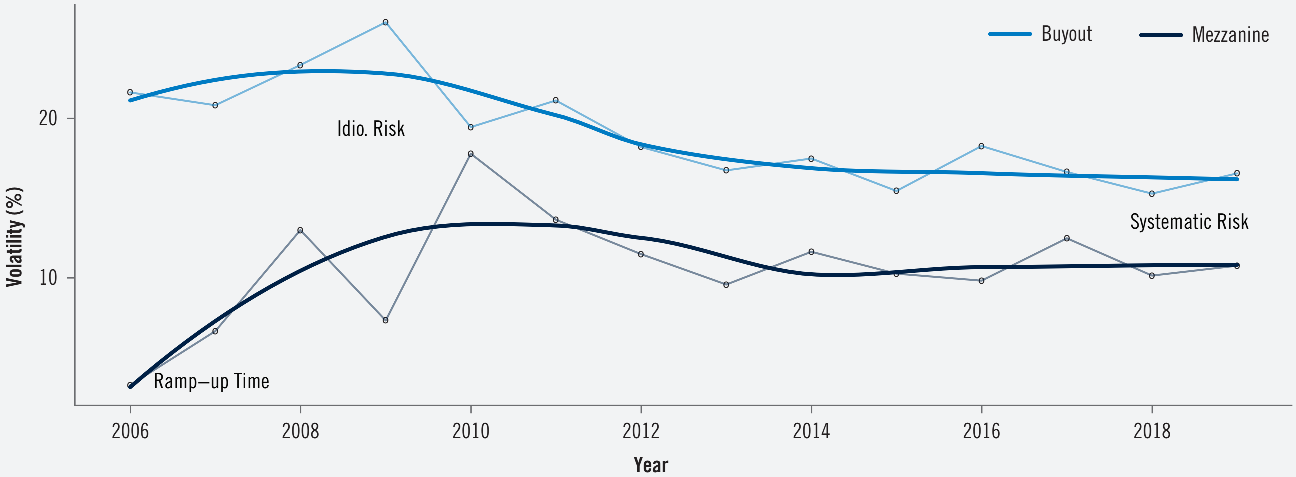
Figure 4: Evolution of Terminal Wealth Dispersion (over time) of Self-contained Private Investments



Note: X- and Y-axes of each private asset are on a different scale; these scales are not shown for clarity.

Source: PGIM IAS, Burgiss and S&P. Provided for illustrative purposes only.

Figure 5: Evolution of Private Investment Portfolio's Volatility Estimate over the Investment Horizon



Note: These are max-min estimates obtained using the Long (1999) method (see footnote 14) and terminal wealth densities from Figure 4.

Source: PGIM IAS, Burgiss and S&P. Provided for illustrative purposes only.

Figure 4 shows the evolution of the distribution of terminal wealth over time for self-contained private asset portfolios. We use the equations above to compute volatility estimates at different points over the investment horizon. These corresponding volatility estimates (Figure 5) display an interesting evolution. For the first three years the estimates fluctuate as it is a ramp-up period for the portfolio with relatively few funds. Volatility rises after the ramp-up period, then declines as a result of fund diversification as the portfolio uses distributions to make commitments to more funds. Eventually, the volatility estimates settle down to the systematic risk which cannot be further diversified away.

The annual volatility estimates for buyout are about 17% and about 10% for mezzanine (Figure 6). These estimates are reasonable as buyout funds are regarded at least as risky as the broad equity market (S&P 500, volatility = 16.7%) and debt (mezzanine) funds are viewed as having an equity market beta less than 1.

Figure 6: Mean and Volatility of Private Investment Portfolio Returns Using Terminal Wealth Dispersion

Subclass of Private Capital Funds	Estimated Mean (Annual)	Estimated Volatility (Annual)
Buyout	11.63%	16.44%
Mezzanine	8.64%	10.70%

Note: These estimates are computed by averaging the volatilities in the last 5 years. See Figure 5.

Source: PGIM IAS, Burgiss and S&P. Provided for illustrative purposes only.

Fair Performance Comparison of Private and Public Investments – Unlevered

We first compare the average annual performance of various private and public investments using traditional metrics. For private investments we report annual returns using pooled quarterly IRRs.[17] For the public markets we use quarterly index returns (no manager alpha or fees). Figure 7 shows that private investments had mean pooled returns using traditional metrics that are a bit higher compared to those calculated using the terminal wealth dispersion of our self-contained private asset portfolios (Figure 6).[18] In contrast, pooled private market volatility was notably lower. The volatility of reported quarterly returns of private investments are generally known to be understated because these returns tend to be smoothed.[19]

Figure 8 summarizes our "corrected" estimates of the mean and volatility of returns for private investments (from Figure 6) as well as public. The corrected estimates for mean returns for private investments are lower than those in Figure 7 using traditional metrics because they reflect the private investor's entire portfolio including the default index investment. On the other hand, the corrected volatilities for private investments have gone up because there is no serial correlation in the periodic returns as they assumed to be i.i.d. Normal.

Figure 7: Annual Returns for Various Investments Using Traditional Quarterly-Linked Returns; 2005–2018

Investment	Market	#Qtrs	Mean (%)	Vol (%)	Min (%)	Max (%)	Mean / Vol
S&P 500	Public	56	8.98	16.66	–37.00	32.39	0.54
LP Buyout Funds (Pooled)	Private	56	14.13	12.37	–23.95	28.26	1.14
LP Mezzanine Funds (Pooled)	Private	56	9.94	6.81	–6.95	21.00	1.46
Leverage Loans (B/BB-rated)	Public	56	5.15	16.36	–28.98	51.39	0.31
10y+ IG Corporates (FltR)	Public	56	2.50	12.37	–21.11	35.35	0.20
10y+ Baa x-Financial Corporates (FltR)	Public	56	3.16	15.65	–24.56	46.80	0.20
10y+ Baa x-Financial Corporates (TR)	Public	56	6.57	11.91	–10.39	31.97	0.55

Note: All numbers are on annual basis.

Source: PGIM IAS, Burgiss, Bloomberg, S&P and Barclays. Provided for illustrative purposes only.

17 These IRRs are point-to-point IRRs (sometimes approximated using modified-Dietz Return formula) from Burgiss and are computed by pooling cash flows and NAVs across all funds from all vintages in each given subclass of funds.

18 For a risk-adjusted comparison, the focus is often to estimate the risk correctly as the time-series of returns is known. However, in case of private markets, even the time-series of returns is not known precisely. The point-to-point IRRs or modified-Dietz returns are only approximations.

19 Private equity funds can hold investments at cost which results in a smoothed return series which makes these investments appear far less risky.

Figure 8: Annual Returns for Various Investments Using Terminal Wealth Dispersion-Based Returns (for Privates); 2005–2018

Investment	Market	#Qtrs	Mean (%)	Vol (%)	Min (%)	Max (%)	Mean / Vol
S&P 500	Public	56	8.98	16.66	–37.00	32.39	0.54
LP Buyout Funds (Portfolio)	**Private**	**56**	**11.63**	**16.44**	**–50.18**	**70.10**	**0.71**
LP Mezzanine Funds (Portfolio)	**Private**	**56**	**8.64**	**10.70**	**–30.07**	**44.31**	**0.81**
Leverage Loans (B/BB-rated)	Public	56	5.15	16.36	–28.98	51.39	0.31
10y+ IG Corporates (FltR)	Public	56	2.50	12.37	–21.11	35.35	0.20
10y+ Baa x-Financial Corporates (FltR)	Public	56	3.16	15.65	–24.56	46.80	0.20
10y+ Baa x-Financial Corporates (TR)	Public	56	6.57	11.91	–10.39	31.97	0.55

Note: All numbers are on annual basis.

Source: PGIM IAS, Burgiss, Bloomberg, S&P and Barclays. Provided for illustrative purposes only.

Fair Performance Comparison

We now walk through the steps of our fair comparison methodology.

We first compare the performance of different investments assuming they are levered to match the *ex post* volatility of buyout investments. Comparing the performance based on the ratio of their mean return and volatility implicitly assumes that low-volatility strategies can be levered up at no cost. However, a leveraged investment incurs financing costs, not just interest (e.g., 3m LIBOR) but also a financing spread and haircut.

Figure 9 presents the financing and cost assumptions used in the levered return calculations (as well as manager alphas and fees for public investments – discussed below).[20] Note: No investment is levered more than 1x.

We recognize that our *ex post* analysis, with the accounting of leverage costs, is not replicable because it assumes we know at the outset the realized *ex post* volatility. For proper *ex ante* analysis to lever up a low-volatility strategy one must correctly predict both current and target volatilities.[21]

Figure 9: Manager Alpha, Fees and Cost of Debt Financing for Leverage – Assumptions

Investment	Manager Alpha (bp)	Financing Spread (bp)	Manager Fees (bp)	Interest	Haircut
S&P 500	50 (100)	50	15	3mL	0.1
LP Buyout Funds	—	—	—	—	—
LP Mezzanine Funds	—	100	—	3mL	0.25
Leverage Loans (B/BB-rated)	100 (200)	100	45	3mL	0.25
10y+ IG Corporates (FltR)	50 (100)	50	15	3mL	0.1
10y+ Baa x-Financial Corporates (FltR)	50 (100)	50	15	3mL	0.1
10y+ Baa x-Financial Corporates (TR)	50 (100)	50	15	3mL	0.1

Note: All numbers are annualized and are specified in basis points. To generate variable manager alpha, we sample Normally distributed alpha from the mean and standard deviation (in bracket) specified above.

Source: PGIM IAS, Burgiss, Bloomberg, S&P and Barclays. Provided for illustrative purposes only.

20 Example calculation: $R_{lev} = (R_{unlev} + \alpha - fees)\times L_h - (int + spd)\times(L_h - 1);\ L_h = 1 + (L-1)\times(1-h);\ L = \frac{Vol_{buyout}}{Vol}$.

21 Estimating volatility from a given time series is difficult because volatility is not constant over time. Predicting volatility is an even harder problem as one must often rely on estimates based on historical returns, and for the first few periods one cannot compute or predict volatility. Consequently, *ex ante* risk-adjusted analysis of levered returns can only be performed after enough periodic returns have been observed to estimate volatilities. In this paper we stick to *ex post* analysis.

Figure 10: Analysis of Annual Returns of Various Investments – Corrected Levered, with no Alpha, Fees or Financing Costs 2005–2018

Investment	Market	#Qtrs	Mean (%)	Vol (%)	Min (%)	Max (%)	Mean / Vol
S&P 500	Public	56	9.83	16.45	–40.63	36.32	0.60
LP Buyout Funds (Portfolio)	Private	56	11.63	16.44	–50.18	70.10	0.71
LP Mezzanine Funds (Portfolio)	Private	56	11.52	15.89	–45.87	64.43	0.73
Leverage Loans (B/BB-rated)	Public	56	8.12	16.52	–46.47	91.55	0.49
10y+ IG Corporates (FltR)	Public	56	3.61	16.54	–37.77	67.14	0.22
10y+ Baa x-Financial Corporates (FltR)	Public	56	4.61	16.51	–38.67	80.10	0.28
10y+ Baa x-Financial Corporates (TR)	Public	56	10.75	16.60	–19.80	58.57	0.65

Note: All numbers are on annual basis.
Source: PGIM IAS, Burgiss, Bloomberg, S&P and Barclays. Provided for illustrative purposes only.

Figure 11: Analysis of Annual Returns of Various Investments – Corrected Levered, with Alpha, Fees and Financing Costs 2005–2018

Investment	Market	#Qtrs	Mean (%)	Vol (%)	Min (%)	Max (%)	Mean / Vol
S&P 500	Public	56	10.92	16.42	–38.70	37.26	0.66
LP Buyout Funds (Portfolio)	Private	56	11.63	16.44	–50.18	70.10	0.71
LP Mezzanine Funds (Portfolio)	Private	56	11.52	15.89	–45.87	64.43	0.73
Leverage Loans (B/BB-rated)	Public	56	8.55	16.40	–44.43	91.95	0.52
10y+ IG Corporates (FltR)	Public	56	3.98	16.60	–37.83	69.47	0.24
10y+ Baa x-Financial Corporates (FltR)	Public	56	5.01	16.55	–38.65	82.32	0.30
10y+ Baa x-Financial Corporates (TR)	Public	56	11.15	16.54	–19.93	60.94	0.67

Note: All numbers are on annual basis.
Source: PGIM IAS, Burgiss, Bloomberg, S&P and Barclays. Provided for illustrative purposes only.

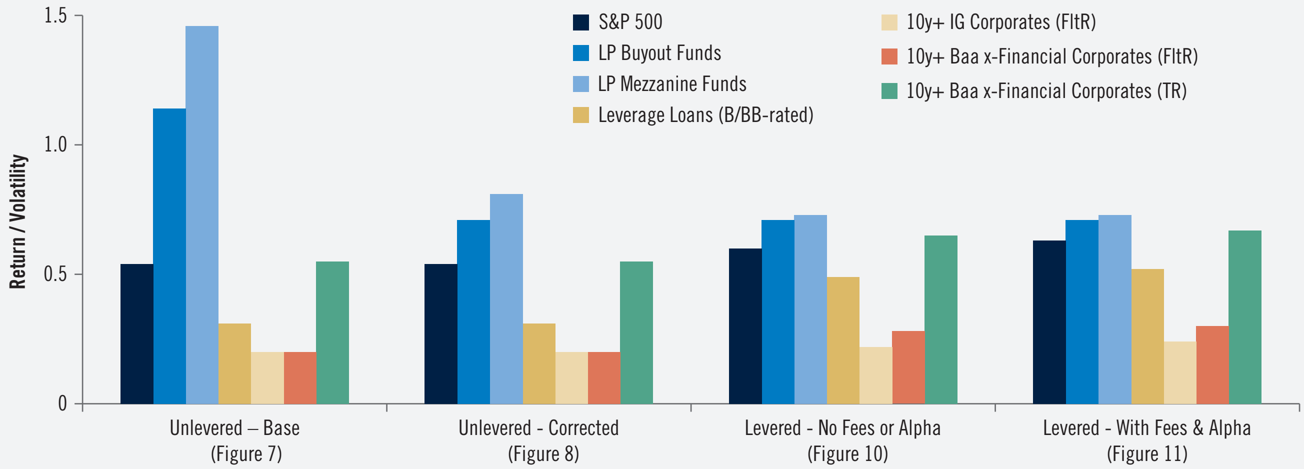
Figures 10 compares different investments on a risk-adjusted basis but ignores manager alphas, fees and haircut and only considers the interest (3mL) that must be paid on the debt for leverage.

Our final step in our fair comparison methodology adds financing spread and haircut as well as public asset alpha and fees. Financing spread and haircut lower the mean return and can prevent enough leverage to match the targeted volatility (note: the volatility of interest rates gets factored into the volatility of levered returns). We also add a variable alpha and fixed fees to the public index returns. Private asset returns include the value of GP skill (net of fees), so for a fair comparison we add a reasonable level of alpha and fees to the public index returns. Figure 11 contains the results.

Figure 12 plots the Mean/Vol ratio from Figures 7, 8, 10 and 11 for easy comparison. On a fair comparison basis we see that levered investment in mezzanine was very competitive with buyout. The 10y+ Baa x-Financial Corporates total return index when levered was also comparable to private markets. The other investment strategies in the public markets were not as attractive compared to private markets performance but may be useful from liquidity, risk and diversification perspectives.[22]

22 The comparison is sensitive to the choice of investment horizon, we chose to work with a period from 2005-2018, which is considered a low-interest rate era.

Figure 12: Comparison of Risk-Adjusted Annual Returns, Various Investments 2005–2018



Note: All numbers have been annualized.

Source: PGIM IAS, Burgiss, Bloomberg, S&P and Barclays. Provided for illustrative purposes only.

Conclusion

We develop a methodology to compare various investments in public and private markets on a risk-adjusted basis – a fair comparison framework. However, risk estimation for private market investment using traditional methods is fraught with challenges. To get around these we use a terminal wealth dispersion-based method to estimate the volatility of returns experienced by a typical private capital portfolio. Our method only requires long-term, not periodic, fund-level performance data (IRR and TVPI) to estimate risk. The volatility of private market strategies using our methodology are well estimated as their time series settles down to what may be considered their systematic risk. We compare our "corrected" risk estimates for private markets strategies (buyout and mezzanine) with those for several public market investments.

Several questions remain to be explored. For example, our analysis is *ex post*. Ideally, we need a method to leverage a low-volatility strategy *ex ante* to match the *ex post* volatility of a high-volatility strategy. This is a nontrivial challenge as the volatility estimates for high-volatility strategies change over time and chasing a changing volatility may result in something completely undesired. Among other unexplored questions are an alternative to synthetic cash flows to get around the issue of risk underestimation and smarter methods to invert terminal wealth dispersion. Perhaps a cash-flow model such as Takahashi and Alexander (2002) can be used in place of synthetic cash flows.

Finally, we plan to explore the impact of a private investor's LP fund selection skill, *i.e.*, an LP's ability to identify good GPs to commit too. Given evidence of GP performance persistence, an LP's selection skill is an important source of portfolio performance. Most LPs would insist that they do have fund-selection skill that must be incorporated in a fair comparison framework.

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